

OYOP: Your 2 essay(s) must be TYPESET, and Double or Triple spaced. Use the *Print/Revise* cycle to produce good, well thought out, essays. Start each essay on a *new* sheet of paper.

Due: **By noon, on Friday, 07Dec2012**, slid *completely* under my office door. Then please email me.

E1: Use \mathbb{D}_N for the set of derangments in \mathbb{S}_N , and let $d_N := |\mathbb{D}_N|$. Generalize and say that d_{-1}, d_{-2}, \dots are each zero. (Reasonable, as we can view \mathbb{S}_{-1} as being empty.)

For natnum K , let $\mathbb{F}_N(K)$ be the set of permutations in \mathbb{S}_N which have precisely K many fixed-points; so $\mathbb{F}_N(0)$ is \mathbb{D}_N . And $\mathbb{F}_3(7)$ is empty.

a Derive and prove a formula for

$$e_N(K) := |\mathbb{F}_N(K)|.$$

Can you give two different proofs?

b Let $P_n(K)$ be the probability that a random element of \mathbb{S}_n has precisely K fixed-points. Compute $\lim_{n \rightarrow \infty} P_n(K)$.

c Can you generalize any part of this problem in a mathematically-interesting way?

E2: **i** Over a 31 day month, Combinatorist Cathy posts at least one soln per day, for a total of 38 solns. PROVE:

There is a period of consecutive days over which she posted exactly $g := 24$ solutions.

[g for "Guaranteed".] NOTE: In your proof, let s_n denote the number of solns posted that month by the end of day n . By hyp., then,

$$1 \leq s_1 < s_2 < \dots < s_{31} = 38.$$

Let $t_n := 24 + s_n$. Using this notation, write a complete, rigorous proof, proving any lemmas you need/want. [Hint: You may find it easier to first show that $g=23$ is guaranteed. Then you'll see how to show that $g=24$ is guaranteed.]

ii Generalize: Replace 31 by D , replace 38 by P ; we now consider posints with $D < P$. Give a formula for the largest

value, call it $\Gamma(D, P)$, for which your proof *guarantees* the values $g = 1, 2, 3, \dots, \Gamma(D, P)$.

iii For fixed D and P , let $\mathcal{M}(D, P)$ be the set of guaranteed posints g . What can you tell me about the structure of $\mathcal{M}(D, P)$? Conjectures? Proofs? Computer experiments? (I don't know the structure. What can you teach me?)

End of Individual Project-E

E1: _____ 105pts

E2: _____ 140pts

Total: _____ 245pts

Please PRINT your name and ordinal. Ta:

Ord:

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HONOR CODE: "I have neither requested nor received help on this exam other than from my professor."

Signature:

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