Advanced Calc
MAA4102 & MAA5104

Home-E

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This Delightful Exam is due **2PM, Friday, 08Dec2006**, slid **under** my LIT402 door.

Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

Fill in *ALL blanks*; neat pencil is fine.

E1: Show no work.

1 $\log_B(64) = \frac{3}{5}$, where B= ______.

$\log_{\sqrt{243}}(27)$= ______.

2 For $0 \leqslant t < 1$: $\int_0^{70+t} [x - \lfloor x \rfloor]\, dx$= ______.

[*Hint:* Graph the integrand fnc.]

3+ Set $H(t) := \exp(t^2)$. Centered at zero, its Taylor series is $\sum_{n=0}^{\infty} B_n t^n$. Numbers B_2= ______, B_4= ______.

Let p_n be the polynomial st. $H^{(n)} = p_n \cdot H$. There is a simple update formula, $p_{n+1} = \text{Formula}(p_n, p_n')$, where $p_{n+1}(t)$= ______.

E2: a Let $f(x) := \frac{\log(x)}{x}$. So $f'(x)$= ______ and $f''(x)$= ______. Determine (no proof) these seven open intervals: $\text{Dom}(f)$=(___ , ___).

$\{f<0\}$=(___ , ___) & $\{f>0\}$=(___ , ___).

$\{f'<0\}$=(___ , ___) & $\{f'>0\}$=(___ , ___).

$\{f''<0\}$=(___ , ___) & $\{f''>0\}$=(___ , ___).

So $\text{MaxPt}(f)$= ______ and $\text{InflexionPt}(f)$= ______.

b Use the above info to make a full-page LARGE graph of f, with all above points and intervals labeled. (Use brightly different colors, so that I can understand what you are "talking about". Sugg: Carefully graph f once, make photocopies and use colors to mark the data for f on one photocopy, the data for f' on another, f'' on another...)

c The Heart of the Problem: For posreals y and z, I want you to study this relation:

1: $$y^z = z^y, \quad \text{with } y < z.$$

How is (1) connected to properties of the above f?

Prove that if both $y, z > 5$, then (1) has **no solns**. But 5 is too big a lower-bound for this: What is the correct (i.e, lowest) lower-bound $\mathbf{L}$= ______ so that

2: *For all $y, z > \mathbf{L}$, relation (1) has **no solns**.*

d Now prove that for each $z > \mathbf{L}$, *there exists* a **unique** posreal y less than $\mathbf{L}$, such that (1). Visually, you'll want to use Graph(f) in your argument so that you can mark the corresponding (y, z)-pairs.

To **prove** rigorously the existence of a y corresponding to z, you'll want to use the IVT.

To **prove** rigorously that there is only one y corresponding to a given z, you'll likely want to use something about f' in order to show strict monotonicity of f in the interval where you need it.

e You know my admonition: "*I always ask that you do more than what I ask you to do.*" What else can you say for this problem? Can you parameterize

$$y(s) := \text{Formula}(s) \quad \text{and} \quad z(s) := \text{Formula}(s)$$

the pairs? If not, what about a specific (y, z)-pair? What posreals $y < \mathbf{L}$ that have **no** corresponding z?

E3: A Consider a cts $F\colon J \to \mathbb{R}$, where $J := [3, 7]$. If $\int_J F \cdot g = 0$ for each bnded RI (Riemann integrable) fnc g, **prove** that $F() \equiv 0$.

B Contrast: Replace "cts" by "bnded and RI", and *produce* an F (nec, discts) where the conclusion fails.

E4: A diff'able $H\colon \mathbb{R}_+ \to \mathbb{R}$ has $\lim_{t\to\infty} [H + H'](t) = 7$. Use L'H to **prove** that these next two limits exist, with values $\lim_{t\to\infty} H(t) = 7$ **and** $\lim_{t\to\infty} H'(t) = 0$. [*Hint:* Apply L'H to ratio $\frac{f \cdot H}{f}$, for a cleverly-chosen fnc f.]

E1:	___ ___ ___	110pts
E2:	___ ___ ___	135pts
E3:	___ ___	95pts
E4:	___ ___	95pts
Total:	___ ___ ___	435pts

Print name ______ Ord:

Honor Code: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ______