Staple!

Ord: ___ ___

Calc 2
MAC2312

Home-E

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Touch: 18Mar2017

This is an **Optional** (see email) take-home project to be done *individually*. (Although it can raise or lower your grade, a decent effort, that is complete, will likely raise your grade.) This is due by **4PM, Friday, 23Apr**. *Fill-in* every *blank* on this sheet. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797···. Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$.

For the essay question, TYPESET *triple–spaced, grammatical, solutions. A free math-typesetting program is* LaTeX*; it is customizable to individual preference.*

E1: Define fncs g, h on the punctured-reals $\mathbb{R} \setminus \{0\}$ by $g(x) := \exp(\frac{1}{x^2}) = \mathrm{e}^{1/x^2}$, and $h(x) := 1/g(x)$. Let

$$\mathcal{L}(x) \quad := \quad \begin{Bmatrix} 0\,, & \text{if } x = 0 \\ h(x)\,, & \text{if } x \neq 0 \end{Bmatrix}.$$

I'd like you to compute the Maclaurin series of $\mathcal{L}$ in several steps.

i Use a full sheet of paper to draw a large labeled graph of $\mathcal{L}()$. State all symmetries you see in the graph. How many points-of-inflection does $\mathcal{L}$ have, and where are they? (This will help you graph $\mathcal{L}$.) Where are the maxima/minima of $\mathcal{L}$? What vert/hor asymptotes does it have? (Show all this on your labeled graph.)

Use l'Hôpital's rule to carefully show the following. *For each natnum K:*

1: $$\lim_{x\to 0} \frac{h(x)}{x^K} \quad = \quad 0\,.$$

Now show: *For each polynomial $R()$, necessarily*

2: $$\lim_{x\to 0} \big[R(\tfrac{1}{x})\cdot h(x)\big] \quad = \quad 0\,.$$

ii Given a polynomial P, define

3: $$f_P(x) \quad := \quad \begin{Bmatrix} 0\,, & \text{if } x = 0 \\ P(\frac{1}{x})\cdot h(x)\,, & \text{if } x \neq 0 \end{Bmatrix}.$$

(So f_1 is simply the $\mathcal{L}$ fnc.) Compute the derivative, $f'_P(x)$, by handling the x=0 case directly from the **defn** of derivative (you'll use the above l'Hôpital's result).

Give a nice **formula**, in terms of P, for a new polynomial $\widehat{P}$ satisfying that $\boxed{[f_P]' = f_{\widehat{P}}}$. (You'll want the above l'Hôpital's result.)

iii At this juncture, let Q_0 be the constant-1 polynomial. Inductively define $Q_{k+1} := \widehat{Q_k}$, for each $k \in \mathbb{N}$. Exhibit a nice table showing the coefficients of polynomial Q_k, for $k = 1, 2, 3, 4$, at least. (Align vertically corresponding powers of x. Make the table nice and clear.) (Do you see a pattern, or a recurring property, here?)

You have established that $\mathcal{L}$ is ∞ly differentiable by showing that $\mathcal{L}^{(k)} = f_{Q_k}$, for each $k = 0, 1, 2, \ldots$. Now compute the Maclaurin series of $\mathcal{L}$;

$$\Lambda(x) \quad := \quad \sum_{n=0}^{\infty} \frac{\mathcal{L}^{(n)}(0)}{n!} \cdot x^n\,.$$

Graph $\Lambda()$. What relation does it have with $\mathcal{L}()$? What does this Mac-series tell you about the graph of $\mathcal{L}()$ in part (i)?

iv Do something extra, involving $\mathcal{L}$ and these nice ideas.

E2: *Henceforth, show no work. Simply fill-in each blank on the problem-sheet.*

a Let $h := [y \mapsto \cos(2y)]$. Then the 5-topped poly $\mathbf{T}^h_{5,0}(x)$= ______________________.

ii Writing poly $p(x) := 5 - 3x + 77x^2 + 9x^3 + 5x^4$ as $\sum_{k=0}^{4} C_k\cdot[x-1]^k$, coeff C_3 is in: Circle *one* interval
(-∞, -70), [-70, -15), [-15, -8), [-8, -1), [-1, 8), [8, 15), [15, 30), [30, 75), [75, 94), [94, +∞).

End of Home-E

E1:	___ ___ ___	000pts
E2:	___ ___ ___	000pts
Un- or poorly stapled:	___	−5pts
Total:	___	0pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

______________________ Ord: ___ ___