

**E1:** Short answer. Show no work.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE**  $\neq \{\}$   $\neq 0$ .

10 **a** The answer is:

*Only one, but the light bulb has to want to change.*

The question is: (See Canvas.)

20 **b** On a 4-set, there are \_\_\_\_\_ many equivalence relations.

20 **c** Let  $\mathcal{P}_\infty$  denote the family of all *co-finite* subsets of  $\mathbb{N}$ . That is, a subset  $S \subset \mathbb{N}$  is an *element* of  $\mathcal{P}_\infty$  IFF  $\mathbb{N} \setminus S$  is finite. Define relation  $\bowtie$  on  $\mathcal{P}_\infty$  by:  $A \bowtie B$  IFF  $A \cap B$  is infinite.

Stmt “This  $\bowtie$  is an equivalence-relation” is: T F

25 **d** Euler  $\varphi(36300) = 2^A \cdot 3^B \cdot 5^C \cdot 7^D \cdot 11^E$ , where  $A=$  \_\_\_\_\_,  $B=$  \_\_\_\_\_,  $C=$  \_\_\_\_\_,  $D=$  \_\_\_\_\_,  $E=$  \_\_\_\_\_ are in  $\mathbb{N} = \{0, 1, 2, 3, \dots\}$ .

As a single number,  $\tau(36300) =$  \_\_\_\_\_.

20 **e** Number  $p := 347$  is prime. Each integer is mod- $p$  congruent to one value in  $J := [-173..173]$ . Use Wilson’s theorem to (easily!) compute  $V \in J$ , where  $V \equiv_p 343!$ , the factorial of 343. This  $V=$  \_\_\_\_\_.

*Essay.* Hand-write your essay on paper, writing large and dark. [You may typeset it if you wish, but that is likely slower.] *Do Not Scrunch!* You can write on every 2<sup>nd</sup>-line to make your essay easier to read. The essay is written in complete sentences, correctly spelled and punctuated, and assembled into logical paragraphs.

When the essay is done, convert it to a pdf, then name it as follows: **E2.<your name>.pdf**

E.g, Rachel Stein will name her pdf file as **E2.Rachel-Stein.pdf**

There are no spaces in the filename! The extension is “pdf”. A hyphen is used to separate the given-name from the family-name. (You may write names in the order you are accustomed.)  $\square$

55 **E2:** Give a *formal statement* of Wilson’s Thm, fully quantified.

Give a careful, detailed *formal proof* of Wilson’s Thm, in complete grammatical sentences. [The 2<sup>nd</sup> part should start with “Proof of Wilson’s thm:”, and then define the necessary objects/notions.]

10 **E3:** “I have neither requested nor received help on this exam other than from my professor.”

**E1:** \_\_\_\_\_ 95pts

**E2:** \_\_\_\_\_ 55pts

**E3:** \_\_\_\_\_ 10pts

**Total:** \_\_\_\_\_ 160pts