

Combinatorics
MAD4204 4563

Class-E

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Welcome. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. **Write expressions unambiguously e.g. “ $1/a + b$ ” should be bracketed either $[1/a] + b$ or $1/[a + b]$. (Be careful with negative signs!)**

Recall $[x \downarrow K] := x \cdot [x - 1] \cdot [x - 2] \cdots [x - [K - 1]]$ is read as “ N falling-factorial K ”.

E5: Short answer. Show no work.

a Prof. King believes that writing in complete, coherent sentences is crucial in communicating Mathematics, improves posture, and whitens teeth. Circle one:

True! Yes! wH’at S a?sEnTENCe

b The physics lab has atomic *zinc, tin, silver* and *gold*. I’m allowed to take 7 atoms, so I have [expressed as single integer] many possibilities.

c ITOf binomial coeffs, the n^{th} Catalan-# is $c_n =$
And $c_5 =$

d For posint N , let G_N be K_N (complete graph) but with one edge removed (so G_N has N vertices). Its chromatic polynomial is $\mathcal{P}_{G_N}(x) =$
[You may use rising/falling factorial in your answer, if you wish.]
In particular, $\mathcal{P}_{G_4}(x) =$

e Writing $1/[1 + 2x]^8$ as $\sum_{n=0}^{\infty} a_n \cdot x^n$, then $a_5 = M \cdot \binom{U}{D}$
where $M =$, $U =$, $D =$

f Suppose $G(x)$ is the OGF of seq. $\vec{p} = (p_0, p_1, \dots)$, where p_n is the number of partitions of n whose parts are primes < 9 .
Then $G(x) =$
[You may leave $G(x)$ as a product of rational fncs.]

OYOP: In grammatical English **sentences**, write your essay on every **third** line (usually), so that I can easily write between the lines. Please number the pages “1 of 57”, “2 of 57” ... (or “1/57”, “2/57”...) I suggest you put your name on each sheet.

E6: Give a formal definition of b_n , the n^{th} **Bell number**, defining any terms you need.

Compute, $b_4 =$

Showing the details, in complete grammatical sentences, derive the EGF of \vec{b} .

End of Class-E

E5: 100pts

E6: 30pts

Total: 130pts

HONOR CODE: “I have neither requested nor received help on this exam other than from my professor (or his colleague).”
Name/Signature/Ord

Ord: