Staple!

Ord: ___ ___

DiffyQ MAP2302	**Individual E-Optional Project**	Prof. JLF King *Touch*: 6May2016

Due no-later-than: noon, Friday, 12Dec. slid *completely* under my office door, LITTLE HALL 402 [top floor, north-east corner.] Then please *email me* that you have handed-in a project.

Your essay must be TYPESET, and double or triple spaced. Use the *Print/Revise* cycle to produce a clear, logically structured, essay.

Write expressions unambiguously e.g, "$1/a + b$" should be bracketed either $[1/a] + b$ or $1/[a + b]$. (Be careful with negative signs!)

Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$.

For the *Laplace transform* of f, use $\mathcal{L}(f) = \widehat{f}$.

E1: *How does an ice cube melt?*

Your essay should have many large, well-drawn pictures, of the various situations.

Terms to research/look-up. ***"Specific heat"*** of a substance; in the sense of "heat capacity per unit mass of a material". ***"Heat of fusion"*** and ***"latent heat"*** of a substance as it melts from solid to a liquid at the same temperature. ***"Thermal conductivity"*** of a substance. For those of these quantities that you need, give letter-names to them. It is possible that the values differ for ice and for water [so you may need two names]. *Use the simplification* that thermal conductivity is independent of temperature.

You may need the density of water, and the density of ice; if so, give them letter names. Or perhaps you only need a name for the *ratio* of their densities.

Carefully define any other terminology that you need; give citations to Wikipedia pages, or elsewhere, where appropriate. ASIDE: Both you and I may use tmp to abbreviate "temperature".

Ice. Use ***hunk*** for the ice "cube" [shapes will be discussed in a moment] under discussion.

A hunk at tmp $0°$ [all tmps are in centigrade] is suspended[♡1] in a bath of water at teMperature $\boldsymbol{\mathcal{M}} > 0°$.

As the hunk melts, assume that it keeps its shape [a ball remains a ball; a cube remains a cube] as its volume, $V(t)$, decreases. Let $\boldsymbol{\mathcal{V}}_0$ denote the initial volume.

Water-bath of ∞ volume. Suppose that there is so much water that its tmp does not appreciably change as the ice melts. The water is well-mixed, so that as soon as some $0°$-ice phase-transitions ["melts"] into $0°$-water, that them there water is whisked away and does not affect the hunk's melting. [Not to worry; that water's tmp will be raised to $\boldsymbol{\mathcal{M}}$ by the water-bath.]

a Using *letters* [which you have defined in a table] for the various ice&water constants and initial-values, write the DEs for $V(t)$ both in the case that the hunk is a ball, and is a cube. Number these DEs as (1_{Ball}) and (1_{Cube}).

b If you can solve these DEs [does SoV work?], then do so. Compute the ***meltdown time*** [the time for the hunk to completely melt into water] symbolically.

c Now substitute in the actual values for ice&water for, say, an ice ball-or-cube of the size used in a drink. Look up the actual melting times [or do the experiment yourself with real ice]; how does your result compare to actual times?

Water-bath of finite volume. Let $\boldsymbol{\mathcal{W}}_0$ be the initial volume of water. Let $W(t)$ denote the volume of water at time t; so $W(t)$ increases with t, while $V(t)$ decreases. *However*, the sum

$$\text{Vol}(t) \quad := \quad V(t) + W(t)$$

is not constant, as ice is less-dense than water [which is why ice floats in water].

The ice is always at $0°$. With $\boldsymbol{\mathcal{M}}_0$ the initial water tmp, let $M(t)$ denote the water's tmp at time t.

d In this model, the water's tmp decreases, as the ice melts. Note also that, now, as the ice melts, the newly-extant water needs to absorb enough heat-energy to become the temperature of the ambient water [the tank is well-mixed].

[♡1] Since we don't want the complexity of the hunk floating and touching a surface of the container, you may imagine this entire experiment takes place on the Space Station, in 0-gravity. Or that some force repels the hunk from the container surface.

Using letters, write down the system of DEs for the

$$V(t),\ W(t),\ M(t)$$

triple, in the case that the hunk is indeed a cube.

e Come up with some exact or approximate soln to this system. You might try a power-series approach, and approximate $V(t)$, $W(t)$, $M(t)$ by polynomials.

Alternatively, can you compute numerical approximations via some method? And/or, at the initial value $(\mathcal{V}_0, \mathcal{W}_0, \mathcal{M}_0)$, can you approximation the DEs by linear DEs, to at least get the melting-rate at time $t = 0$?

E2: i Suppose that $P(), Q(), G()$ are $\mathbf{C}^\infty$-fncs, and $y = y(t)$ is a soln to DE

$$*: \qquad y'' + P(t)y' + Q(t)y = G(t).$$

Argue carefully that y *must* be a $\mathbf{C}^\infty$-fnc.

ii Assume that there is an analytic soln $y = y(t)$ to IVP

$$**: \qquad y'' + y' + y = \cos(10t),$$

with $y(0) = 0$ and $y'(0) = 1$. Compute the first three *non-zero* terms of the Taylor polynomial approximation [centered at 0] of $y(t)$.

Individual
End of E-Optional
Project

E1:	___ ___ ___	000pts
E2:	___ ___ ___	000pts
Total:	___	0pts

Please PRINT your name and ordinal. Ta:

Ord:

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HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature:

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