

Differential Eqns
MAP2302

D-Practice

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Touch: 17Nov2017

Hi. Whatever you do, *Don't Panic!* This freebie practice is light-years longer than the actual exam.

Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. **Write expressions unambiguously** e.g, “ $1/a + b$ ” should be bracketed either $[1/a] + b$ or $1/[a + b]$. (Be careful with negative signs!)

Do **not** approx.: If your result is “ $\sin(\sqrt{\pi})$ ” then write that rather than .9797...

Use “ $f(x)$ notation” when writing fncs; in particular, for trig and log fncs. E.g, write “ $\sin(x)$ ” rather than the horrible $\sin x$ or $[\sin x]$.

Notation. For the Laplace transform of f , use $\mathcal{L}(f) = \hat{f}$. Use \mathcal{L}^{-1} for the inVerse Laplace-transform operator. Use **H()** for the *Heaviside fnc*, also called the unit-step fnc. And use $\delta()$ for the *Dirac delta “function”*.

D1: Show no work.

a With $\mathbf{1}()$ the constant-1 fnc and $F(x) := \sin(5x)$, then, convolution

$$[\mathbf{1} \otimes F](x) = \dots$$

b With $\mathbf{1}()$ the constant-1 fnc and $F(x) := e^{2x}$, then, convolution

$$[\mathbf{1}^{\otimes 4} \otimes F](x) = \dots$$

c With $f(x) := e^{7x}$ and $g(x) := e^{4x}$, then

$$[f \otimes g](5) = \dots$$

d Matrices A, B, U are 2×2 , with U is invertible.

Then $e^{A+B} = e^A \cdot e^B$: AT AF Nei

$Ue^B U^{-1} = e^{UBU^{-1}}$: AT AF Nei

If e^B invertible, then B is invertible: AT AF Nei

e Fncs $x(t)$ and $y(t)$ satisfy this system of DEs,

$$\begin{aligned} x' + x - 3y &= 0, \\ y' + 6x - 8y &= 0. \end{aligned}$$

It can be written as $Y' = M \cdot Y$, where $Y := \begin{bmatrix} x \\ y \end{bmatrix}$ and M is matrix

Characteristic poly of M is $\wp_M(z) = \dots$

A soln has $x(t)$ a linear combination of $e^{\alpha t}$ and $e^{\beta t}$ for numbers $\alpha = \dots$ and $\beta = \dots$

f Matrix $G := \begin{bmatrix} 2 & -1 & 3 \\ 4 & -2 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

is nilpotent. Computing, $G^2 = \dots$

The $(1, 3)$ -entry of e^{Gt} is \dots

g We can re-write function

$$f(t) := \dots \cos\left(\frac{3}{4}\pi + 5t\right) + \sqrt{2} \cdot \cos\left(\frac{3}{2}\pi + 5t\right)$$

as $f(t) = R \cdot \cos(\theta + 5t)$, for **real** numbers

$$R = \dots \geq 0 \text{ and } \theta = \dots \in [0, 2\pi).$$

h Let $B := \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$, $M := \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$. and $R := MBM^{-1}$.

Then

$$e^{Rt} = \dots$$

i The Laplace transform of fnc $f(t) := \cos(7t)$ is

$$\hat{f}(s) = \dots$$

For IVP $3y'' - y = \cos(7t)$ with $y(0)=2$ and $y'(0)=5$, then,

$$\hat{y}(s) = \dots$$

j $\mathcal{L}(t^{26}e^{3t})(s) = \dots$

$$\mathcal{L}(\sin(2t) \cdot \exp(3t))(s) = \dots$$

Determine the inverse-transform, please.

$$\mathcal{L}^{-1}\left(\frac{3s + 5}{s^2 + 2s + 5}\right)(t) = \dots$$

k Suppose $y(0) = 2$, $y'(0) = 3$, $y''(0) = 5$. Then $\mathcal{L}(y^{(3)} + 2y')(s)$ equals $[[p(s) \cdot \hat{y}(s)] + q(s)]$ for **polynomials**

$p(s)=$ _____
 and $q(s)=$ _____

OYOP: In grammatical English **sentences**, write your essay on every **third** line (usually), so that I can easily write between the lines.

D2: i Start your essay with this sentence-fragment, and complete the defn using as many sentences as you need:

Saying that function $f:[0, \infty) \rightarrow \mathbb{R}$ has “exponential order 5”, i.e. $f \in \text{Ord}(5)$, means that...

ii Give one example of a continuous fnc $g:[0, \infty) \rightarrow \mathbb{R}$ which lies neither in $\text{Ord}(1)$, nor in $\text{Ord}(2)$, nor in $\text{Ord}(3)$... I.e, this fnc g violates *every* exponential order.

D3: Give a *careful* argument, that $\mathcal{L}(f \otimes g)$ equals $\widehat{f} \cdot \widehat{g}$.

D4: Let $g(t) := \mathbf{H}(t - 5) \cdot f(t - 5)$. Give a *careful* argument, that $\widehat{g}(s)$ equals $e^{-5s} \cdot \widehat{f}(s)$.

D5: i Start your essay with this sentence-fragment, and complete the defn using as many sentences as you need:

An $N \times N$ matrix B is nilpotent if... Moreover, saying that its nilpotency degree is 4 means that...

ii Give an example of 3×3 matrix which has nilpotency degree 2.

End of D-Practice