Staple!

Team: ______

Numbers & Polys
MAS3300

Home-D

Prof. JLF King
Touch: 21Feb2017

Note. Write expressions unambiguously e.g, "$1/a + b$" should be bracketed either $[1/a] + b$ or $1/[a + b]$. (Be careful with **negative** signs!)

D1: Show no work.

__ __ z) Professor King sometimes gives freebie questions. Circle one: **True** **What a guy!** ***Who?***

__ __ a) The finite set $\mathcal{P}(\mathcal{P}(\{\text{3-Stooges}\}))$ has cardinality ....

__ __ b) For $G := \{1, 2, 3, 4\}$, consider $f: G \to \mathcal{P}(G)$ by

$$f(1) := \{3, 4\}, \qquad f(2) := G,$$
$$f(3) := \varnothing, \qquad f(4) := \{1, 4\}.$$

The set $B := \{x \in G \mid f(x) \not\ni x\}$ is $\{$ .... $\}$.

__ __ c*) Compute the $\beta \in [0..176)$ st. $15 \cdot \beta \equiv 1 \pmod{176}$. The Euclidean Alg. yields that $\beta=$ ....

__ __ d*) In let x be the rational number $0.2\overline{16}$. Write $x = \frac{p}{q}$ for co-prime posints $p=$ .... , $q=$ .... [*Hint:* Look at the hint after (4.29).]

__ __ e*) Let $\alpha := \sqrt{3} + \sqrt{7}$. Redoing the computation of class, $x^4 +$ ...... $x^3 +$ ...... $x^2 +$ ...... $x +$ ...... is a quartic intpoly having α as a root.

Typed essay questions. Recall that every "**if**" must be matched by a "**then**".

D2: Define an open and half-open interval $U := (0, 1)$ and $H := (0, 1]$. Give, with proof, an *explicit* **bijection** $g: U \hookrightarrow\!\!\!\to H$ of the form

$$g(x) := \begin{cases} \text{............}, & \text{if } x \in S; \\ \text{............}, & \text{if } x \in \text{..................}; \end{cases}$$

where the denumerable subset $S := \left\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\right\}$ is a "Cantor's Hotel" inside of U. For your g, what is $g(\frac{2}{3})=$ ...... ? Now use a full sheet of paper to make a large labeled *accurate* **graph** of g. [*Hint:* Your g will necessarily be discontinuous.]

D3: Here A, B, C, D, E are sets. Let C^D be the set of maps $f: D \to C$. (The exponent is the Domain.) Give, with proof, an *explicit* **bijection**

$$\Phi: A^{D \times E} \to [A^D]^E.$$

D4: Carefully write a formal proof that there exist irrational positive reals w, z so that w^z is *rational*, as follows: Let

$$S := \sqrt{7}, \quad T := \sqrt{2} \qquad \text{and} \qquad P := S^T.$$

Argue that either S^T is such an example, or P^T is.

End of Home-D

D1:	__ __ __	150pts
D2:	__ __	70pts
D3:	__ __	70pts
D4:	__ __	70pts
Total:	__ __ __	360pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ....

Ord: ....

Ord: ....

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