Staple!

Team:_____

Moda
MAA4227 MAA5229

Home-D

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Touch: 6May2016

Carefully TYPE, *double-or-triple–spaced, grammatical essays solving the three problems. This is due at* **BoC on Friday, 11Feb2011**.

Please follow the CHECKLIST *on our* Teaching *page.*

Definitions. Say that two integers p and q (not both zero) are ***relatively prime***, written $p \perp q$, if $\text{Gcd}(p, q)$ is 1. A rational $\frac{p}{q}$ is in ***LCTerms*** (lowest common terms) if $p,q \in \mathbb{Z}_+$ with $q > 0$, and $p \perp q$.

A rational number x is a ***dyadic rational*** if, in LCTerms, it is $p/2^n$, with $p \in \mathbb{Z}$ and n a natnum. Thus each integer k is dyadic, since $k = k/2^0$. Also $\frac{-17}{32}$ and $\frac{5}{10}$ are dyadic rationals, but $\frac{1}{10}$ is not, and neither is π nor $\sqrt{32}$. Use ***cty*** and ***discty*** to abbreviate "continuity" and "discontinuity".

Suppose that B is a set of reals (such as an interval) and we have a function $f:B \to \mathbb{R}$. Let $\text{Cty}(f)$ denote the set of $x \in B$ at which f is continuous. Let $\text{DisCty}(f)$ denote the set of $x \in B$ at which f is *dis*continuous.

For a set $S \subset \mathbb{R}$, let $\mathbf{1}_S$ denote the **"*indicator function* of S"**: For $x \in \mathbb{R} \smallsetminus S$, then, $\mathbf{1}_S(x) := 0$. And $\mathbf{1}_S(x) := 1$, for x in S.

D1: Let $\mathbb{D} \subset \mathbb{Q}$ be the set of *dyadic rationals*. Define the ***mountain function*** $M:\mathbb{R}\to\mathbb{R}$ by:

$$M(x) := \begin{Bmatrix} \frac{1}{q} & \text{if } x \in \mathbb{D} \text{ and } x = \frac{p}{q} \text{ in LCTerms;} \\ 0 & \text{if } x \in \mathbb{R} \smallsetminus \mathbb{D}. \end{Bmatrix}.$$

Thus $M(0) = 1$ and $M(\frac{-6}{64}) = \frac{1}{32}$ and $M(\frac{1}{3}) = 0$ and $M(\boldsymbol{\pi}) = 0$.

Prove that M is discontinuous at x IFF x is a dyadic rational. In particular, for each dyadic P, give a **"*witness* of discty of $M()$ at P"**. I.e, give a *particular* positive number $\varepsilon := \text{Formula}(P)$ and sequence $\mathbf{x} := \text{Formula}(P)$, such that

$$\forall n: \quad \left|M(x_n) - M(P)\right| \geqslant \varepsilon\,,$$

yet $\lim(\mathbf{x})$ equals P.

D2: Prove that the mountain fnc <u>*is*</u> R.I on $[0, 1]$, with $\int_0^1 M = 0$, by: Given $\varepsilon>0$, produce (with rigorous proof) a $\delta>0$ so that each pptn P of $[0, 1]$ with $\text{Mesh}(\mathsf{P}) < \delta$ satisfies that $\text{RS}^M(\mathsf{P}) < \varepsilon$.

D3: If $g:\mathbb{R}\circlearrowleft$ is differentiable with $g'(3)=7$, then g is strictly-incr on some open interval about 3. *T* *F*

End of Home-D

D1:	___ ___	95pts
D2:	___ ___	65pts
D3:	___ ___	45pts
Total:	___ ___ ___	205pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ....

Ord: ....