Staple!

Geometry
MTG3214 03H9

Individual-D

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Touch: 17Nov2017

OYOP: Your 2 essay(s) must be TYPESET, and Double or Triple spaced. Use the *Print/Revise* cycle to produce good, well thought out, essays. Start each essay on a *new* sheet of paper. Do **not** restate the problem; just solve it.

Due: By noon, on Friday, 27Apr2012, slid *completely* under my office door. Then please email me.

D1: A ***boomerang*** is a non-convex quadrilat(eral); call its [$>\pi$] interior-angle "thick". Conversely, a quadrilat with each angle $\leqslant\pi$ (a "thin" angle) is a ***kite***. A dissection of a polygon **P** into *finitely many* quadrilats is a "***quadritiling*** of **P**". [Tiles in a quadritiling *need not* be congruent to each other.]

1: Boom-Kite Theorem. *Each quadritiling, $\mathcal{T}$, of a convex polygon **P** must use at least one kite.* ◊

Useful concepts. Let $V_{\mathbf{X}}$ and $V_{\mathbf{I}}$ denote the number of external/internal $\mathcal{T}$-vertices. (So **P** is an $V_{\mathbf{X}}$-gon.) Let K and B be the number of kites/boomerangs in $\mathcal{T}$. (So $N := |\mathcal{T}| = K+B$.) Examine relations among $V_{\mathbf{X}}, V_{\mathbf{I}}, K$ and B. Recall: *"I always ask that you do more than what I ask you to do."* You may want $\mathsf{E} = \mathsf{E}(\mathcal{T})$, the number of edges in the tiling. □

___ ___ **i** For a quadritiled **P**: Prove that $V_{\mathbf{X}}$ is even. [*Hint:* Sum the interior-angles of the $\mathcal{T}$-quadrilats in two ways.]

___ ___ **ii** For a quadritiled convex **P**: Prove that $K \geqslant 1$. Go further: STATE, then PROVE, a stronger, quantitative, thm.

___ ___ **iii** A ***1penta-tiling*** of a polygon, **Q**, uses 1 pentagon, and remaining tiles are quadrilats.

2: Possibly Erroneous Statement. *Each 1penta-tiling, $\mathcal{F}$, of a convex polygon **Q** must have at least one convex tile.* ◊

Analyse your proof of (ii), then provide a proof or a CEX to the "PES".

Note. Use $\mathbf{0} = (0,0) = \left[\begin{smallmatrix}0\\0\end{smallmatrix}\right]$ for the zero-vector. Use ***ITOf*** for "in terms of".

D2: Uncle Euclid inscribed his soln to the Boomerang Problem *on* a crystal boomerang, which he hid. His testament contained instructions designed so that only a talented Geometer could find the boomerang:

Climb the Platonic Plateau and stand at the magestic olive tree, τ. Walk to the column-of-Granite, Γ, counting your paces. Turn right $90°$, walk the same number of paces, and hammer into the ground a granite spike, g.

Return to the tree. Stride to the column-of-Obsidian, Ω, counting your paces. Turn left $90°$, walk this number of paces, and hammer into the ground an obsidian spike, ω. The boomerang will be found halfway between the spikes.

You climb the plateau, see the two columns, but the Olive Tree, τ, has rotted away. Nonetheless. you find the crystal boomerang. *How?*

a The spikes, $g(\tau)$ and $\omega(\tau)$, are functions of the (unknown) vector τ. Compute the matrix

$\mathsf{B} = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$ so that $\mathsf{B}\Gamma = g(\mathbf{0})$. And matrix

$\mathsf{C} = \begin{bmatrix} \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ \end{bmatrix}$ so that $\mathsf{C}\Omega = \omega(\mathbf{0})$.

b ITOf τ, B, Γ, and vector addition/subtraction and matrix multiplication, $g(\tau) =$ ________________.

ITOf τ, C and Ω, our $\omega(\tau) =$ ________________.

c The boomerang is at $\frac{1}{2}[g(\tau)+\omega(\tau)]$, which simplifies to ________________, ITOf $\Gamma, \Omega, \mathsf{B}, \mathsf{C}$, *but no* τ.

d Coordinatizing the plateau, we find that $\Gamma = (3,9)$ and $\Omega = (4,2)$. So Boomerang= (______ , ______).

End of Individual-D

D1:	___ ___ ___	125pts
D2:	___ ___	95pts
Total:	___ ___ ___	220pts

Please PRINT your name and ordinal. Ta:

________________________________ Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ________________________________