

DiffyQ
MAP2302

Individual
D-Optional
Project

Prof. JLF King
Touch: 17Oct2017

This IOP is due **noon, Thurs, 23Apr2015**, slid *completely* under my office door, 402 LITTLE HALL. [top floor, north-east corner.]

This sheet is “Page 1/ N ”, and you’ve labeled the rest as “Page 2/ N ”, . . . , “Page N/N ”. Fill-in [large handwriting] *on this problem-sheet* all of your blanks.

Write **DNE** in a blank *if* the described object does not exist or if the indicated operation cannot be performed.

Your essay must be TYPESET, and double or triple spaced. Use the Print/Revise \odot cycle to produce a clear, logically structured, essay.[3]

Write expressions unambiguously e.g., “ $1/a + b$ ” should be bracketed either $[1/a] + b$ or $1/[a + b]$. (Be careful with negative signs!)

Use “ $f(x)$ notation” when writing fncs; in particular, for trig and log fncs. E.g, write “ $\sin(x)$ ” rather than the horrible $\sin x$ or $[\sin x]$.

For the Laplace transform of f , use $\mathcal{L}(f) = \hat{f}$. Do **not** restate the problem; just solve it.

D1: *How does an ice cube melt?*

Your essay should have many large, well-drawn pictures, of the various situations.

Terms to research/look-up. “*Specific heat*” of a substance; in the sense of “heat capacity per unit mass of a material”. “*Heat of fusion*” and “*latent heat*” of a substance as it melts from solid to a liquid at the same temperature. “*Thermal conductivity*” of a substance. For those of these quantities that you need, give letter-names to them. It is possible that the values differ for ice and for water [so you may need two names]. Use the *simplification* that thermal conductivity is independent of temperature.

You may need the density of water, and the density of ice; if so, give them letter names. Or perhaps you only need a name for the *ratio* of their densities.

Carefully define any other terminology that you need; give citations to Wikipedia pages, or elsewhere, where appropriate. ASIDE: Both you and I may use **tmp** to abbreviate “temperature”.

Ice. Use *hunk* for the ice “cube” [shapes will be discussed in a moment] under discussion.

A hunk at tmp 0° [all tmps are in centigrade] is suspended^{♥1} in a bath of water at temperature $\mathcal{M} > 0^\circ$.

As the hunk melts, assume that it keeps its shape [a ball remains a ball; a cube remains a cube] as its volume, $V(t)$, decreases. Let \mathcal{V}_0 denote the initial volume.

Water-bath of ∞ volume. Suppose that there is so much water that its tmp does not appreciably change as the ice melts. The water is well-mixed, so that as soon as some 0° -ice phase-transitions [“melts”] into 0° -water, that them there water is whisked away and does not affect the hunk’s melting. [Not to worry; that water’s tmp will be raised to \mathcal{M} by the water-bath.]

a Using *letters* [which you have defined in a table] for the various ice&water constants and initial-values, write the DEs for $V(t)$ both in the case that the hunk is a ball, and is a cube. Number these DEs as (1_{Ball}) and (1_{Cube}) .

b If you can solve these DEs [does SoV work?], then do so. Compute the *meltdown time* [the time for the hunk to completely melt into water] symbolically.

c Now substitute in the actual values for ice&water for, say, an ice ball-or-cube of the size used in a drink. Look up the actual melting times [or do the experiment yourself with real ice]; how does your result compare to actual times?

Water-bath of finite volume. Let \mathcal{W}_0 be the initial volume of water. Let $W(t)$ denote the volume of water at time t ; so $W(t)$ increases with t , while $V(t)$ decreases. *However*, the sum

$$\text{Vol}(t) := V(t) + W(t)$$

is not constant, as ice is less-dense than water [which is why ice floats in water].

The ice is always at 0° . With \mathcal{M}_0 the initial water tmp, let $M(t)$ denote the water’s tmp at time t .

^{♥1}Since we don’t want the complexity of the hunk floating and touching a surface of the container, you may imagine this entire experiment takes place on the Space Station, in 0-gravity. Or that some force repels the hunk from the container surface.

d In this model, the water's tmp decreases, as the ice melts. Note also that, now, as the ice melts, the newly-extant water needs to absorb enough heat-energy to become the temperature of the ambient water [the tank is well-mixed].

Using letters, write down the system of DEs for the

$$V(t), W(t), M(t)$$

triple, in the case that the hunk is indeed a cube.

e Come up with some exact or approximate soln to this system. You might try a power-series approach, and approximate $V(t), W(t), M(t)$ by polynomials.

Alternatively, can you compute numerical approximations via some method? And/or, at the initial value $(\mathcal{V}_0, \mathcal{W}_0, \mathcal{M}_0)$, can you approximation the DEs by linear DEs, to at least get the melting-rate at time $t = 0$?

D2: **i** Suppose that $P(), Q(), G()$ are \mathbf{C}^∞ -fncs, and $y = y(t)$ is a soln to DE

$$*: \quad y'' + P(t)y' + Q(t)y = G(t).$$

Argue carefully that y must be a \mathbf{C}^∞ -fnc.

ii Assume that there is an analytic soln $y = y(t)$ to IVP

$$**: \quad y'' + y' + y = \cos(10t),$$

with $y(0) = 0$ and $y'(0) = 1$. Compute the first three *non-zero* terms of the Taylor polynomial approximation [centered at 0] of $y(t)$.

D3: From our textbook, write an essay solving problem #34 on P.411. This problem discusses integrating the n^{th} -derivative of the Dirac-delta against a sufficiently differentiable function f .

Individual
End of D-Optional
Project

D1: ___ ___ ___ 000pts

D2: ___ ___ ___ 000pts

D3: ___ ___ ___ 000pts

Total: ___ 0pts

Please PRINT your *name* and *ordinal*. Ta:

Ord:

.....

HONOR CODE: "I have neither requested nor received help on this exam other than from my professor."

Signature:

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Folks, I have had a great time working with you this Semester. Stop by next semester to "Talk Math".

Cheers, Prof. Sieve-brain