Staple!

Ord: ___ ___

Combinatorics
MAD4203 3214

Proj-D

Prof. JLF King
01Dec2017

D1:	___ ___ ___	000pts
D2:	___ ___ ___	000pts
D3:	___ ___ ___	000pts
Total:	___	0pts

Welcome. Due by 2:30PM, Thur., 07Dec 2017, *slid completely under my office door*, Lit402.

Pamphlet *Chromatic polynomial of a graph* on our COMB COURSE PAGE has defns. Please use the notation from that pamphlet. Use **u**━**v** to mean that vertices **u** and **v** have an edge between them. □

D1: Produce two (finite) simple graphs, A, B and posints q, r st.

$$\mathcal{P}_A(q) > \mathcal{P}_B(q) \quad \textit{yet} \quad \mathcal{P}_A(r) < \mathcal{P}_B(r)$$

D2: For natnum J and tuple $\vec{\mathbf{b}} = (b_1, b_2, \dots, b_J)$ of posints, a simple graph is $\vec{\mathbf{b}}$-***nice*** if its chromatic-poly is

$$*: \quad f_{\vec{\mathbf{b}}}(x) := x \cdot [x-1]^{b_1} \cdot [x-2]^{b_2} \cdots [x-J]^{b_J}\,.$$

Let $\mathcal{W}$ be the set of nice graphs.

a) Give an algorithm which, given a $\vec{\mathbf{b}}$, explicitly constructs a $\vec{\mathbf{b}}$-nice graph $S_{\vec{\mathbf{b}}}$.

The number of edges in $S_{\vec{\mathbf{b}}}$ is $\sum_{n=1}^{J}$ ....

b) When $J := |\vec{\mathbf{b}}| = 1$, prove that each $\vec{\mathbf{b}}$-nice graph, H, is a tree.

c) Prove or CEX: "*Collection $\mathcal{W}$ is sealed under* (basic) *full-product.*"

d) Prove or CEX: "*Each nice graph with $J \geqslant 2$, is chromatically unique.*"

e) Can you characterize the nice graphs?

D3: Prove there is *no* graph S whose chromatic polynomial is

$$h(x) := x \cdot [x^5 - 5x^4 + 12x^3 - 10x^2 + 3x - 1]\,,$$

using results from our *Chromatic polynomial* pamphlet, and from Bona's text.

End of Proj-D

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

Ord: ....

Folks, I've had a great time learning Combinatorial material together. See you next semester, starting with Generating-functions and continuing with Graph Theory. (Please read our text during the break.) *Looking way ahead, stop by in future semesters for Math/chess/coffee/frisbee.*

Cheers, Prof. (Per)PLEX-*ed*