Staple!

Ord: ___ ___

Plex
MAA4402 2838

IndividualOP-D

Prof. JLF King
Thur, 13Apr2017

This IOP (Indiv. Optional Project) is due **2PM, Thur., 20Apr2017**, slid *completely* under my office door, 402 LITTLE HALL. This sheet is "Page 1/*N*", and you've labeled the rest as "Page 2/*N*"..."Page *N*/*N*".

Abbrevs. Let SCC mean "positively oriented simple-closed-contour". For a SCC C, have $\mathring{C}$ be the (open) region C encloses, and let $\widehat{C}$ mean C together with $\mathring{C}$. So $\widehat{C}$ is $C \cup \mathring{C}$; it is automatically simply-connected and is a closed bounded set.

D1: Short answer. Show no work.
Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

___ z In ball $\mathrm{Bal}_1(0)$, there are ____ solutions to

$$2z^9 - z^6 - 6z^3 + z \;=\; 1\,. \quad [\textit{Hint:}\ \text{Rouche's thm.}]$$

___ a For $N \geqslant 2$, number $V_N =$ ____

where $$\int_{-\infty}^{\infty} \frac{1}{x^2 + [N+1]x + N^2}\,\mathrm{d}x \;=\; \frac{2\pi}{V_N}\,.$$

___ b Let $h(z) := \exp(\frac{1}{z}) \cdot \exp(\frac{1}{3z})$. Then $\operatorname{Res}_{z=0}\big(h(z)\big) =$ ____. And $\operatorname{Res}_{z=0}\big(z \cdot h(z)\big) =$ ____.

___ c Consider entire function $G(z) := \sum_{n=0}^{\infty} a_n \cdot [z-8]^n$. Then

$$\operatorname{Res}_{z=0}\Big(\frac{G(z)}{z^4}\Big) \;=\; \sum_{n=K}^{\infty} a_n \cdot W_n\,, \quad \textit{where}$$

$K =$ ____ $\in \mathbb{N}$ and $W_n =$ ____.
[Each W_n is a number. You may use binomial coefficients in expressing your W_n.]

___ d On $B := \mathrm{Bal}_4(0)$, fncs $f(z) :=$ ____ and $g(z) :=$ ____ are analytic, and distinct. Points $p_n :=$ ____ $\in B$ satisfy $f(p_n) = g(p_n)$, yet no contradiction, since $p_n \xrightarrow{n} \mathbf{q} \in \partial B$.

Your essay must be TYPESET, *and double or triple spaced. Use the Print/Revise cycle to produce a clear, logically structured, essay. Do* **not** *restate the problem; just solve it.*

D2: For $N = 4, 5, 6, \ldots$, define annulus

$$\mathbf{A}_N \;:=\; \Big\{ z \in \mathbb{C} \;\Big|\; N < |z| < N{+}1 \Big\}, \quad \textit{and polynomial}$$

$$F_N(z) \;:=\; z^N \;+\; Nz \;+\; [2N^2 + N^N]\,.$$

Polynomial $F_N()$ has ____ roots in $\mathbf{A}_N$.

Prove your result, using Rouche's thm, carefully specifying what contours you are using, giving a detailed, complete argument establishing the inequalities you need.

Provide good, LARGE, *Labeled* pictures of the annulus, the contours and F_N-zeros, at least for N=4 and N=5.

Do something extra: Can you generalize the problem in a mathematically interesting way? Can you give me more information of the locations of the roots, as a fnc of N?

End of IndividualOP-D

D1: ___ ___ ___ 000pts

D2: ___ ___ ___ 000pts

Total: ___ 0pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

Ord: ____

Folks, I've had a great time learning Complex Analysis with you. See if the (Fall 2017) *Combinatorics course interests you. In any case, stop by in future semesters for Math/chess/coffee.*

Cheers, Prof. (Per)PLEX-*ed*