Staple!

**Ord:** ___ ___

Plex
MAA4402 2838

**IndividualOP-D**

Prof. JLF King
Thur, 13Apr2017

This IOP (Indiv. Optional Project) is due **2PM, Thur., 20Apr2017**, slid *completely* under my office door, 402 LITTLE HALL. This sheet is "Page 1/$N$", and you've labeled the rest as "Page 2/$N$"… "Page $N/N$".

**Abbrevs.** Let **SCC** mean "positively oriented simple-closed-contour". For a SCC $C$, have $\mathring{C}$ be the (open) region $C$ encloses, and let $\widehat{C}$ mean $C$ together with $\mathring{C}$. So $\widehat{C}$ is $C \cup \mathring{C}$; it is automatically simply-connected and is a closed bounded set.

**D1:** Short answer. Show no work.
Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

___ ___ z In ball $\text{Bal}_1(0)$, there are ________ solutions to

$$2z^9 - z^6 - 6z^3 + z = 1. \quad [\textit{Hint: } \text{Rouche's thm.}]$$

___ ___ a For $N \geqslant 2$, number $V_N =$ ________________________

where

$$\int_{-\infty}^{\infty} \frac{1}{x^2 + [N+1]x + N^2}\,dx = \frac{2\pi}{V_N}.$$

___ ___ b Let $h(z) := \exp(\frac{1}{z}) \cdot \exp(\frac{1}{3z})$. Then $\operatorname{Res}_{z=0}(h(z)) =$ ____________. And $\operatorname{Res}_{z=0}(z \cdot h(z)) =$ ____________.

___ ___ c Consider entire function $G(z) := \sum_{n=0}^{\infty} a_n \cdot [z-8]^n$.
Then

$$\operatorname{Res}_{z=0}\left(\frac{G(z)}{z^4}\right) = \sum_{n=K}^{\infty} a_n \cdot W_n, \quad \text{where}$$

$K =$ ______ $\in \mathbb{N}$ and $W_n =$ ________________________.

[Each $W_n$ is a number. You may use binomial coefficients in expressing your $W_n$.]

___ ___ d On $B := \text{Bal}_4(0)$, fncs $f(z) :=$ ______________________ and $g(z) :=$ ______________________________ are analytic, and distinct. Points $p_n :=$ ______________________ $\in B$ satisfy $f(p_n) = g(p_n)$, yet no contradiction, since $p_n \overset{n}{\to} \mathbf{q} \in \partial B$.

*Your essay must be* TYPESET, *and double or triple spaced. Use the* Print/Revise *cycle to produce a clear, logically structured, essay. Do* **not** *restate the problem; just solve it.*

**D2:** For $N = 4, 5, 6, \ldots$, define annulus

$$\mathbf{A}_N := \left\{ z \in \mathbb{C} \;\middle|\; N < |z| < N+1 \right\}, \quad \textit{and polynomial}$$

$$F_N(z) := z^N + Nz + [2N^2 + N^N].$$

Polynomial $F_N()$ has ____________________ roots in $\mathbf{A}_N$.

Prove your result, using Rouche's thm, carefully specifying what contours you are using, giving a detailed, complete argument establishing the inequalities you need.

Provide good, LARGE, *Labeled* pictures of the annulus, the contours and $F_N$-zeros, at least for $N=4$ and $N=5$.

Do something extra: Can you generalize the problem in a mathematically interesting way? Can you give me more information of the locations of the roots, as a fnc of $N$?

End of IndividualOP-D

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|---|---|---|
| **D1:** | ___ ___ ___ | 000pts |
| **D2:** | ___ ___ ___ | 000pts |
| **Total:** | ___ | 0pts |

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
*Name/Signature/Ord*

________________________________________________ *Ord:* ___ ___

*Folks, I've had a great time learning Complex Analysis with you. See if the* (Fall 2017) *Combinatorics course interests you. In any case, stop by in future semesters for Math/chess/coffee.*
*Cheers, Prof.* (Per)PLEX*-ed*