Sets and Logic
MHF3202

Class-D

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D1: Short answer. Show no work.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

15 **a** On a 3-set, there are ______ many equiv.relations.

15 **b** Let $\mathcal{P}_\infty$ denote the family of all *infinite* subsets of $\mathbb{N}$. Define relation $\approx$ on $\mathcal{P}_\infty$ by: $A \approx B$ IFF $A \cap B$ is infinite. Stmt "*This $\approx$ is an equivalence-relation*" is: *T F*

20 20 **c** A K-set Ω has ______ **non-symmetric** binrels. Its number of **anti-symmetric** binrels is ______

[**Note:** Do not confuse symmetric with reflexive. Be *careful* on this problem.]

10 15 **d** Suppose that $\prec$ is a total-order on set $\mathcal{S}$, and $<$ is total-order on set $\boldsymbol{\Omega}$, both strict. Define binrel $\lll$ on $\mathcal{S} \times \boldsymbol{\Omega}$ by:

$$(b, \beta) \lll (c, \gamma)$$

IFF *Either* $b \prec c$ *or* $[b = c$ and $\beta < \gamma]$.

Then:

Relation $\lll$ is a total-order. *T F*

Suppose $\prec$ and $<$ are each well-orders.

Then $\lll$ is a well-order. *T F*

Essay. Hand-write your essay on paper, writing large and dark. [You may typeset it if you wish, but that is likely slower.] *Do Not Scrunch!* You can write on every 2^{nd}-line to make your essay easier to read. The essay is written in complete sentences, correctly spelled and punctuated, and assembled into logical paragraphs.

When the essay is done, convert it to a pdf, then name it as follows: `D2.<your name>.pdf`

E.g, Rachel Stein will name her pdf file as

`D2.Rachel-Stein.pdf`

There are no spaces in the filename! The extension is "`pdf`". A hyphen is used to separate the given-name from the family-name. (You may write names in the order you are accustomed.) □

10 45 **D2:** For $K = 0, 1, 2, \ldots$, define sum

$$\mathcal{S}_K := \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{K \cdot [K+1]}$$

$$\stackrel{\text{note}}{=} \sum_{n=1}^{K} \frac{1}{n \cdot [n+1]} .$$

Find a closed-form [no summation sign, nor dot-dot-dot] for $\mathcal{S}_K$. Prove your formula correct by induction on K.

10 **D3:** *"I have neither requested nor received help on this exam other than from my professor."*

D1:	___ ___	95pts
D2:	___ ___	55pts
D3:	___ ___	10pts
Total:	___ ___ ___	160pts