Linear Algebra
MAS4105 *3247*

Class-D

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Note. Below, **V** and **W** are VSes, and $\mathsf{T}\colon \mathbf{V}\to\mathbf{V}$ and $\mathsf{S}\colon \mathbf{W}\to\mathbf{W}$ are linear.

D1: a $\mu =$ ______ $\leqslant \nu =$ ______

are the eigenvalues of $\mathsf{G} := \begin{bmatrix} -3 & 21 \\ -2 & 10 \end{bmatrix}$. Let $\mathsf{D} := \begin{bmatrix} \mu & 0 \\ 0 & \nu \end{bmatrix}$. Then $\mathsf{D} = \mathsf{U}^{-1}\mathsf{G}\mathsf{U}$ where the 2×2 *integer* matrix U is

$$\mathsf{U} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}.$$

b Apply Cramer's Rule to write x_1 as a *rational function*,

$x_1 =$ ______, of variables

$A, G, R, T, U, \alpha, \beta, \gamma$, where $\begin{bmatrix} R & U & G \\ 0 & A & 0 \\ 0 & 0 & T \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$.

c The point $P := (5, -2)$, in the plane, has orthogonal projection $Q :=$ (______ , ______) on $\mathbb{L}$, the line $y = 1 + 3x$. [*Check* that $Q \in \mathbb{L}$ and $[P - Q] \perp \mathbb{L}$.]

Essay questions. *Start each question on a new sheet of paper. Do Not restate the problem. Write your sentences on every third line, so that I can easily write between the lines.*

D2: On $\mathbf{V} := \mathbb{R}^9$, these T-eigenvecs $\mathbf{u}_1, \ldots, \mathbf{u}_5$ have *distinct* evals $\alpha_1, \ldots, \alpha_5$. OYOSOP, prove that collection $\mathcal{C} := \{\mathbf{u}_1, \ldots, \mathbf{u}_5\}$ is linearly-indep. *DNRTP!*

D3: Let $\mathsf{B} := \begin{bmatrix} 1 & 2 & 1 & 0 & 1 \\ 3 & 6 & 0 & -3 & 0 \end{bmatrix}$. Then $\mathsf{R} := \mathit{RREF}(\mathsf{B})$ is [show no work, here]

$$\mathsf{R} = \begin{bmatrix} \quad & \quad & \quad & \quad & \quad \end{bmatrix}.$$

I For subspace $\mathbf{V} := \mathrm{Nul}(\mathsf{B})$, use back-substitution, and *scaling*, to produce an *integer basis*

$\mathbf{v}_1 :=$ (, , , ,) $\mathbf{v}_2 :=$ (, , , ,)

$\mathbf{v}_3 :=$ (, , , ,). [Show No Work, here!]

[*Note:* Only use as many as the dimension of **V**.]

II Using *sentences* and *pictures*, explain & show the Gram-Schmidt algorithm computing an **orthogonal** *integer-basis* for **V**:

$\mathbf{b}_1 :=$ (, , , ,), $\mathbf{b}_2 :=$ (, , , ,)

$\mathbf{b}_3 :=$ (, , , ,). [Entries are integers]

Arrange that the Gcd of the entries in each vector is 1, and that the first non-zero value is positive. (Do *not* show computation of inner-products.)

End of Class-D

D1:	___ ___	90pts
D2:	___ ___ ___	110pts
D3:	___ ___	90pts
Total:	___ ___ ___	290pts

Please PRINT your name and ordinal. Ta:

Ord:

______

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ______