Combinatorics 1
MAD4203

Class-D

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D1: Show no work. Write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

Recall that a perm of cycle-type $\lceil 7^3, 4^2, 1^8 \rfloor$ has three 7-cycles, two 4-cycles, and eight fixed-points. Use ***CN*** for "cycle notation", and ***CCN*** for "canonical CN".

a) Perm $\beta \in \mathbb{S}_{15}$ has sig $\lceil 5^3 \rfloor$. It has ______ many sqroots with sig $\lceil 5^3 \rfloor$, and ______________________ with sig $\lceil 10^1, 5^1 \rfloor$.

b) Perm $\pi := [6,7,8,1,3,2,4,5]$ has $\mathrm{Sgn}(\pi) = +\mathbf{1}\ -\mathbf{1}$.

c) In $\mathbb{S}_{13}$, the maximum possible order of an element is $\mathrm{MaxOrd}(\mathbb{S}_{13}) = \mathrm{Lcm}($ ________________ $) =$ ______ .

OYOP: *In grammatical English **sentences**, write your essays on every **third** line* (usually), *so that I can easily write between the lines.*

D2: i) For natnums N, K, define $c(N,K)$, the **"*signless Stirling number* of the first kind"**.

ii) Prove: THM: *For posints $N \geqslant K$, recurrence*

$$c(N,K) \;=\; [N-1]\cdot c(N-1,K) \;+\; c(N-1,K-1)$$

holds.

iii) Let $H_N(x) := \prod_{j=0}^{N-1}[x+j]$.
Prove: LEMMA: *For each posint N,*

$$\sum_{k=0}^{N} c(N,k)\cdot x^k \quad = \quad H_N(x)\,.$$

D3: On a 9×9 chessboard, 37 rooks are placed. Prove there exists a ***friendly*** 5-set of rooks. [I.e, on 5 distinct rows and on 5 distinct columns.] [*Hint:* PHP] Illustrate the concepts in your proof with *large, useful Pictures.*

[*Hint:* The rooks are the pigeons, but what are the pigeon-holes?]

End of Class-D

D1:	___ ___	85pts
D2:	___ ___ ___	105pts
D3:	___ ___	75pts
Total:	___ ___ ___	265pts

Please PRINT your name and ordinal. Ta:

Ord: ____

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ____