Combinatorics 1
MAD4203

Class-D

Prof. JLF King
Touch: 30Sep2019

D1: Show no work. Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0 \neq$ *Empty-word*.

A perm of cycle-signature $\lceil 7^3, 4^2, 1^8 \rfloor$ has three 7-cycles, two 4-cycles, and eight fixed-pts. Use ***CN*** for "cycle notation", and ***CCN*** for "canonical CN".

a) Perm $\beta \in \mathbb{S}_{15}$ has sig $\lceil 5^3 \rfloor$. It has ____ many sqroots with sig $\lceil 5^3 \rfloor$, and ____ with sig $\lceil 10^1, 5^1 \rfloor$.

b) Perm $\pi := [6,7,8,1,3,2,4,5]$ has $\mathrm{Sgn}(\pi) = +1\ -1$.

c) In $\mathbb{S}_{13}$, the maximum possible order of an element is $\mathrm{MaxOrd}(\mathbb{S}_{13}) = \mathrm{LCM}($ ____ $) =$ ____.

OYOP: *In grammatical English sentences, write your essays on every 2nd line* (usually)*, so that I can easily write between the lines.*

D2: i) For natnums N, K, define $\mathbf{c}(N,K)$, the "***signless Stirling number*** of the first kind".

ii) Prove: THM: *For posints $N \geqslant K$, recurrence*

$$\mathbf{c}(N,K) \ = \ [N-1] \cdot \mathbf{c}(N-1,K) \ + \ \mathbf{c}(N-1,K-1)$$

holds.

iii) Let $H_N(x) := \prod_{j=0}^{N-1} [x+j]$.
Prove: LEMMA: *For each posint N,*

$$\sum_{k=0}^{N} \mathbf{c}(N,k) \cdot x^k \quad = \quad H_N(x)\,.$$

D3: On a 9×9 chessboard, 37 rooks are placed. Prove there exists a ***friendly*** 5-set of rooks. [I.e, on 5 distinct rows and on 5 distinct columns. Shorthand: You may use ***clump*** for "friendly 5-set".] Illustrate the concepts in your proof with *large, useful Pictures*. [*Hint:* PHP]

[*Hint:* The rooks are the pigeons, but what are the pigeon-holes?]

End of Class-D

D1:	___ ___	85pts
D2:	___ ___ ___	105pts
D3:	___ ___	75pts
Total:	___ ___ ___	265pts

Please PRINT your name and ordinal. Ta:

Ord: ____

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ____