Ord: ___ ___

Abstract Algebra
MAS4301 3175

Class-D

Prof. JLF King
Monday, 31Mar2008

D-Home: ___ ___ ___ 275pts

D4: ___ ___ ___ 135pts

Total: ___ ___ ___ 410pts

Hello. Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0 \neq$ *Empty-word*..

Let F and R be the ***flip*** and ***rotation*** in the dihedral group $\mathbb{D}_N$, with $\mathtt{F}^2 = \mathbf{e}$, $\mathtt{R}^N = \mathbf{e}$ and $\mathtt{RFRF} = \mathbf{e}$. Use $\mathtt{R}^j$ and $\mathtt{R}^j\mathtt{F}$ as the standard form of each element in $\mathbb{D}_N$.

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

Ord: ....

D4: Show no work.

___ ___ a) The φ fnc, $\varphi(N) := \#\{k \in [1..N] \mid k \perp N\}$, is named after: Archimedes Euler Fermat Gallian Gauss Klein Lagrange.
And $\varphi(363) =$ ...., AsAPOfPrimePowers.

___ ___ b) Circle the one group which is *not* isomorphic to any of the others:

$\mathbb{Z}_2 \times \mathbb{Z}_6$ $\quad \mathbb{D}_6$ $\quad \mathbf{U}(13)$ $\quad \mathbb{Z}_4 \times \mathbb{Z}_3$ $\quad \mathbb{S}_3 \times \mathbb{Z}_2$.

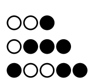
The remaining four groups can be paired into two isomorphic pairs. Underline the cyclic pair.

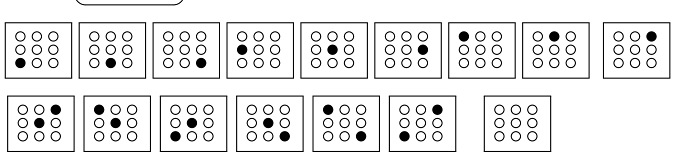
___ ___ c) With $\mathbb{B}$ the 4×4×4 Qubic board, let $\mathbf{\Gamma}$ be its TTT-automorphism group, which has 192 elements. Let $\mathbf{c}, \mathbf{f} \in \mathbb{B}$ be a center and face cell. Then Orb-Stab says

$|\mathrm{Stab}_{\mathbf{\Gamma}}(\mathbf{c})| =$ .... $=$ .... and $|\mathrm{Stab}_{\mathbf{\Gamma}}(\mathbf{f})| =$ .... $=$ ....

___ ___ d) The map $\psi: U(40) \to U(40)$ is a homomorphism with $\mathrm{Ker}(\psi) = \{1, 9, 17, 33\}$ and $\psi(11) = 11$. Then $\psi^{-1}(11) = \{$ .... $\} \subset U(40) \subset [0..40)$.

___ ___ e) The four conjugacy-classes of $\mathbb{D}_5$ are: $\{\mathbf{e}\}$,
$\{$ .... $\}$, $\{$ .... $\}$, $\{$ .... $\}$.

___ ___ f) This peg configuration  is Klein-equivalent to which position in its 3×3 subsquare?:



End of Class-D