Advanced Calc
MAA4102 & MAA5104

Prac-D

Prof. JLF King
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Please. Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$. Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than $.9797\cdots$. Write expressions unambiguously e.g, "$1/a+b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with **negative** signs!)

Abbrevs: **WtSaCi** for "Write the Sentence and Complete it". **G.E.O** for "Give (an) example of". **ITOf** for "in terms of". **st.** for "such that"

Use **nv-** for "non-void", e.g "consider a nv-closed set K". Use **MS** for "metric space". Use **RI** for "Riemann Integrable" or "Riemann Integral".

Use $\overline{\mathbb{R}}$ for $[-\infty,+\infty]$, the "extended reals".

For each of the limit questions, write "$+\infty$", "$-\infty$", a real number, or *–if none of these–* "DNE". Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

D1: Show no work.

a) Suppose $h\colon[3,5)\to\mathbb{R}$ is cts. Then h *uniformly continuous.* *AT AF Nei.*

If $f\colon\mathbb{R}\circlearrowleft$ is diff'able, then f' is cts. *AT AF Nei.*

If $f\colon\mathbb{R}\circlearrowleft$ is diff'able, then f is cts. *AT AF Nei.*

b) Consider a diff'able $h\colon\mathbb{R}\to\mathbb{R}$.

If $|h'|<22$ then h is unif-cts. *AT AF Nei.*

If $|h'|>22$ then h is unif-cts. *AT AF Nei.*

c) $\dfrac{d}{dx}\displaystyle\int_5^{\sin(x^2)}\log =$ ______.

$\dfrac{d}{dx}\displaystyle\int_x^{x+7}\cos(\cos(t))\,dt =$ ______.

d) $\displaystyle\lim_{x\searrow 0}[1+5x]^{2/x}=$ ______.

$\displaystyle\lim_{x\searrow 2}\frac{\sin(x)-x}{x^3}=$ ______.

g) Let $F(x):=\sin(x)^{e^x}$. Its derivative, then, is $F'(x)=$ ______.

D2: G.E.O a $\mathbf{C}^\infty$ fnc on $\mathbb{R}$ which is not analytic at 7:

$$f(x) := \begin{cases} \_\_\_\_ & \text{if } \_\_\_\_ \\ \_\_\_\_ & \text{if } \_\_\_\_ \end{cases}$$

G.E.O a cts fnc on $[-2,5]$ which is not unif-cts.

For $S\subset\mathbb{R}$, use $\mathbf{1}_S$ for the *indicator fnc* of S: Graph $f:=4\cdot\mathbf{1}_{(-\infty,8)}+3\cdot\mathbf{1}_{[5,\infty)}$.

Graph $h(x):=x-\lfloor x\rfloor$.

Essays. On your own sheets of lined paper, give the following definitions or proofs. No "scratch work" accepted, only complete, grammatical, coherent sentences. Write **every 2nd or every 3rd line** for math essays.

D3: State IVT. State MVT. State the Cauchy MVT. State Taylor's thm with Remainder term.

D4: Let $J:=[3,7]$. **WtSaCi**:

A ***pointed partition*** (pptn) $P=(\vec{x},\vec{Q})$ on J is....

Given $f\colon J\to\mathbb{R}$, its ***Riemann Sum*** is $\mathrm{RS}_f(P)$ ....

A fnc $h\colon J\to\mathbb{R}$ is ***Riemann integrable*** IFF ....

D5: Let $g(x):=x^3-3x$. Accurately graph its *positive part* g^+. Accurately graph its *negative part* g^-. Recall that $g^++g^-=|g|$.

D6: Let $f(x):=\sin(2x)$. Its fifth *Taylor polynomial* is $\mathbf{T}_5(x)=$ ______.

D7: **WtSaCi**: On MS (Ω,m), a seq. $\vec{\mathbf{c}}\subset\Omega$ is m-***Cauchy*** IFF ... [*Hint:* Be precise with your quantifiers]

State the Nested Intervals Thm. Prove it.

State the Bolzano-Weierstrass Thm. Prove it.

State the Heine-Borel Thm. Prove it.

D8: On a set Ω, a fnc $\mathsf{m}\colon\Omega\times\Omega\to[0,\infty)$ is a ***metric*** if: $\forall P,Q,R\in\Omega$: (*Write the 3 remaining axioms.*)

D9: Fix a <u>compact</u> MS (Ω,m). a) Prove that each closed subset $E\subset\Omega$ is compact. [*Hint:* The complement, $\Omega\smallsetminus E$, is Ω-open. Use it, together with a given open-cover of E, to produce an open-cover of Ω. Etc.]

b) Prove that Ω is sequentially-compact. [*Hint:* FTSOC, consider a seq. $\vec{\mathbf{s}}\subset\Omega$ with no convergent subseq. Argue, WLOG, that $\vec{\mathbf{s}}$ consists of distinct pts. Now argue that $U:=\Omega\smallsetminus\{s_n\}_{n=1}^\infty$ is open. Now...]

D10: Define seq $\vec{\mathbf{b}}$ by $b_n:=\frac{1}{n\cdot[n+1]}$. Get a closed-formula for $\sum_{j=1}^N b_j=$ ______, for N a posint. One way to do this is to view $\vec{\mathbf{b}}$ as the discrete deriv of ...

End of Prac-D

Print name .... Ord:

Honor Code: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ....