

## Combo class Problems

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### Ladies and Gentlemen –start your engines!

**1.1:  $N$ -towns Theorem.** Consider a network of  $N \geq 1$  towns, each connected to every other town by a one-way<sup>♥1</sup> road. Then . . .

**A:** There exists at-least-one universal town. Town  $\alpha$  is **universal** if for each other town,  $\beta$ , you can legally bicycle from  $\alpha$  to  $\beta$  (possibly passing through intermediate towns).

SOLVED<sub>BY</sub>: John P., 2011t. Zach N., 2012t. Michael E., 2013t. Lizzie [Donna] N-C., 2017g.

**B:** There exists a **2-universal** town; it can access each town using at most two roads [i.e, at most one intermediate town].

SOLVED<sub>BY</sub>: Michael V., Terry T., Alex H., Stephen H., 2011t. Ken D., 2017g.

**C:** In a network of  $N \geq 3$  towns, it is always possible to reverse at most one road so that, now, every town is universal.

SOLVED<sub>BY</sub>: Ken D., 2017g. ◇

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<sup>♥1</sup>We have a **directed graph**; a “**digraph**”. This one is a “**complete digraph** on  $N$  vertices”; it has  $\binom{N}{2}$  directed-edges, that is,  $\frac{1}{2}N[N-1]$  many **oriented edges**.