Staple!

Ord: ___ ___

Differential Eqns
MAP2302

C-Practice

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Notation. For the *Laplace transform* of f, use $\mathcal{L}(f) = \widehat{f}$. Use $\mathcal{L}^{-1}$ for the inverse Laplace-transform operator.

C1: Show no work.

___ ___ a) Function
$y_{\alpha,\beta}(t) =$ ....
is the general soln to

$$y'' + 4y' + 3y \quad = \quad \cos(2t)\,.$$

Also, the *constants* on the LhS are $1, 4, 3$. Suppose the DE describes the position of a spring, with forcing-function $\cos(2t)$. Then the *constant* corresponding to the coefficient-of-friction is ..... .

___ ___ b) With $\mathbf{1}()$ the constant-1 fnc and $F(x) := \sin(5x)$, then, convolution
$[\mathbf{1} \circledast F](x) =$ ....

___ ___ c) With $\mathbf{1}()$ the constant-1 fnc and $F(x) := \mathrm{e}^{2x}$, then, convolution
$[\mathbf{1}^{\circledast 4} \circledast F](x) =$ ....

___ ___ d) With $f(x) := \mathrm{e}^{7x}$ and $g(x) := \mathrm{e}^{4x}$, then
$[f \circledast g](5) =$ ....

___ ___ e) Matrices $\mathsf{A}, \mathsf{B}, \mathsf{U}$ are $2{\times}2$, with U is invertible. Then

$\mathrm{e}^{\mathsf{A}+\mathsf{B}} = \mathrm{e}^{\mathsf{A}} \cdot \mathrm{e}^{\mathsf{B}}$:	*AT*	*AF*	*Nei*
$\mathsf{U}\mathrm{e}^{\mathsf{B}}\mathsf{U}^{-1} = \mathrm{e}^{\mathsf{U}\mathsf{B}\mathsf{U}^{-1}}$:	*AT*	*AF*	*Nei*
If e^{B} *invertible, then* B *is invertible* :	*AT*	*AF*	*Nei*

___ ___ f) Fncs $x(t)$ and $y(t)$ satisfy this system of DEs,

$$\begin{aligned} x' + x - 3y &= 0\,, \\ y' + 6x - 8y &= 0\,. \end{aligned}$$

It can be written as $\mathsf{Y}' = \mathsf{M}{\cdot}\mathsf{Y}$, where $\mathsf{Y} := \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ and M is matrix ....

Characteristic poly of M is $\wp_{\mathsf{M}}(z) =$ ....

A soln has $x(t)$ a linear combination of $\mathrm{e}^{\alpha t}$ and $\mathrm{e}^{\beta t}$ for *numbers* $\alpha =$ .... and $\beta =$ ....

g) Matrix $\mathsf{G} := \begin{bmatrix} 2 & -1 & 3 \\ 4 & -2 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

is nilpotent. Computing, $\mathsf{G}^2 =$ ....

The $(1,3)$-entry of $\mathrm{e}^{\mathsf{G}t}$ is ....

h) We can re-write function

$$f(t) \quad := \quad \cdot\cos\left(\tfrac{3}{4}\pi + 5t\right) + \sqrt{2}\cdot\cos\left(\tfrac{3}{2}\pi + 5t\right)$$

as $f(t) = R{\cdot}\cos\big(\theta + 5t\big)$, for <u>**real**</u> numbers $R =$ .... $\geqslant 0$ and $\theta =$ .... $\in [0, 2\pi)$.

i) Let $\mathsf{B} := \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$, $\mathsf{M} := \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$. and $\mathsf{R} := \mathsf{M}\mathsf{B}\mathsf{M}^{-1}$. Then

$\mathrm{e}^{\mathsf{R}t} =$ ....

j) The Laplace transform of fnc $f(t) := \cos(7t)$ is
$\widehat{f}(s) =$ ....

For IVP $3y'' - y = \cos(7t)$ with $y(0) = 2$ and $y'(0) = 5$, then,

$\widehat{y}(s) =$ ....

k) $\mathcal{L}\big(t^{26}\mathrm{e}^{3t}\big)(s) =$ ....

$\mathcal{L}\big(t^{26} \circledast \mathrm{e}^{3t}\big)(s) =$ ....

$\mathcal{L}\big(\sin(2t){\cdot}\exp(3t)\big)(s) =$ ....

Determine the inverse-transform, please.

$\mathcal{L}^{-1}\left(\dfrac{3s + 5}{s^2 + 2s + 5}\right)(t) =$ ....

Continued...

—— l Suppose $y(0) = 2$, $y'(0) = 3$, $y''(0) = 5$. Then $\mathcal{L}\big(y^{(3)} + 2y'\big)(s)$ equals $\big[[p(s)\cdot\widehat{y}(s)] + q(s)\big]$ for **polynomials**

$q(s)=$ ....

and $p(s)=$ ....

OYOP: *In grammatical English* ***sentences****, write your essay on every* ***third*** *line* (usually)*, so that I can easily write between the lines.*

C2: i Start your essay with this sentence-fragment, and complete the defn using as many sentences as you need:

An $N{\times}N$ ***matrix*** B ***is nilpotent if...*** ***Moreover, saying that its nilpotency degree is*** 4 ***means that...***

—— ii Give an example of $3{\times}3$ matrix which has nilpotency degree 2.

C3: Let $\mathsf{G}(t) := \begin{bmatrix} 0 \\ t \end{bmatrix}$ and $\mathsf{R} := \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, which is nilpotent. U.F. $\mathsf{Z}(t) := \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ satisfies

$$\mathsf{Z}'(t) + \mathsf{R}\cdot\mathsf{Z}(t) \quad = \quad \mathsf{G}(t)\,.$$

Use MacFOLDE to solve for $\mathsf{Z}(t)$, showing all the steps, in complete English sentences. *Each sentence starts with a capitalized* <u>word</u>*, and ends with visible punctuation.*

End of C-Practice