Moda
MAA4226 MAA5228

Individual Project-C

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Carefully TYPE *a triple–spaced, grammatical, essay solving the problem. This final-project is due by* **3PM, Friday, 10 Dec2010**, *slid completely under my office door,* LIT402.

Fill-in every *blank* on this sheet.

Please follow the CHECKLIST on our Teaching page.

C1: For $N{=}1,2,\ldots$, define fncs $R_N, F_N{:}\mathbb{R}{\to}\mathbb{R}$ by

$$R_N(x) := \exp\left(\tfrac{N}{1+Nx^2}\right) \quad\text{and}\quad F_N := 1/R_N\,.$$

___ ___ **a** Prove that, pointwise, $F_1 \geqslant F_2 \geqslant F_3 \geqslant \ldots \geqslant 0$. Use this to prove, $\forall x \in \mathbb{R}$, that $L(x) := \lim_{n\to\infty} F_n(x)$ exists in $\mathbb{R}$. Computing, $L(0) =$ ______ .

___ ___ **b** Let $U := \mathbb{R} \smallsetminus \{0\}$. Define $M{:}U{\to}\mathbb{R}$ by $M := L\lfloor_U$. So $M(x)=$ ______ ; a "Calc-1 fnc".[♡1] Differentiating, $M'(x) = M(x)\cdot\Big[$ ______ $\Big]$.

Using colors, graph L and $F_1,\ldots,F_5$. Indicate horiz. asymptotes, points-of-inflexion (exactly?) and other relevant info.

___ ___ **c** Let $\|\cdot\|$ denote the supremum-norm on functions. Examine/estimate $\|F_n - L\|$. *Prove* or *dis*prove that

$$F_n \xrightarrow[n\to\infty]{\text{uniformly}} L,\ \text{i.e, that}\ \lim_{n\to\infty}\|F_n - L\| \stackrel{?}{=} 0.$$

[For a CEX, give a *specific* $\varepsilon{>}0$, infinite set B of indices, and *specific* points $x_n \in \mathbb{R}$ st. $\forall n \in B$: $\big[F_n - L\big](x_n) \geqslant \varepsilon$.]

___ ___ **d** In any case, prove that $L()$ is continuous at zero. Please prove the.fol proposition, which may help with the above and below tasks. (Is L'Hôpital's Thm useful?)

1: Prop'n. *For each polynomial $P()$, necessarily $\lim_{x\to 0}\big[P(\frac{1}{x})\cdot M(x)\big]$ equals zero.* ◊

With this tool in hand, prove that L is differentiable at zero by directly applying the **definition** (difference-quotient) of derivative.

___ ___ **e** Now prove that L is ∞ly differentiable by establishing the following nifty lemma.

[♡1] Note that $L()$ and $M()$ are *different* functions; after all, they have different domains.

2: Nifty Poly Lemma. *Given a polynomial P, define*

$$\textbf{3:}\qquad J_P\Big(x\Big) := \begin{Bmatrix} 0\,, & \text{if } x = 0 \\ P(\frac{1}{x})\cdot M(x)\,, & \text{if } x \neq 0 \end{Bmatrix}.$$

Prove that J_P is diff'able at the origin (use the **defn** of deriv), *as well as everywhere else. Give a formula* (in terms of P) *for a new poly $\widehat{P}(w)=$* ______

satisfying that $\boxed{[J_P]' = J_{\widehat{P}}}$. (The foregoing Proposition may help in proving equality at zero.) ◊

Let Q_0 be the constant-1 poly. Inductively define $Q_{k+1} := \widehat{Q_k}$, for each $k{\in}\mathbb{N}$. Prove L is ∞ly diff'able by proving $L^{(k)}{=}J_{Q_k}$, for each $k = 0,1,2,\ldots$. E.g., $Q_5(w)=$ ______ .

Exhibit a table showing the coeffs of poly $Q_k(w)$, for $k = 1,\ldots,6$, at least, with powers-of-w aligned. Observed patterns/recurring-properties? *Proofs?*

f Compute the Maclaurin series for $L()$. It has form $\sum_{k=0}^{\infty} C_k\cdot[x-0]^k$ where $C_k{=}$ ______ . Its RoC= ______ . What surprising relation do you note between $L()$ and its Maclaurin series?

Poorly stapled, missing ordinal, name or honor signature:	**C1:**	___ ___ ___	255pts
		___ ___	−15pts
Not double-spaced:		___ ___	−15pts
Poorly proofread:		___ ___	−25pts
Total:		___ ___ ___	255pts

Print name ______ Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ______