DynSys MTG 6401	**Home-C**	Prof. JLF King 12Sep2015

Intro. Due, no later than **4PM,Friday, 23Apr2010**, slid completely under my office door, LIT402.

Below, $(X, \mathcal{X}, \mu)$ is a non-atomic Lebesgue probability space.

C1: i Suppose $(T: X, \mathcal{X}, \mu)$ is weak-mixing, and $S \in C(T)$. If S ergodic, prove that S itself is weak-mixing.

ii Construct a weak-mixing $(T : X, \mathcal{X}, \mu)$ and non-ergodic $S \in C(T)$ st. $S \neq Id$.

C2: Let G be the semigroup of mpts on (X, μ). Given a $B \in \mathcal{X}$, define a pseudo-metric d_B on G by

$$d_B(S, T) \quad := \quad \mu\big(S^{-1}(B) \triangle T^{-1}(B)\big)\,.$$

Let $\vec{\mathbf{B}} = (B_k)_{k=1}^{\infty}$ be a μ-dense family of sets. With $\mathsf{d}_k := d_{B_k}$, define

1: $$\mathsf{m}(S, T) \quad := \quad \sum_{k=1}^{\infty} \tfrac{1}{2^k} \cdot \mathsf{d}_k(S, T)\,,$$

which perforce is a pseudo-metric.

a Prove: PROPOSITION. *If $\mathsf{m}(S, T) = 0$, then $\forall E \in \mathcal{X}$: $d_E(S, T) = 0$. If $\tau_n \xrightarrow{n\to\infty} T$ in (G, m), then $\forall E \in \mathcal{X}$: $d_E(\tau_n, T) \to 0$.*

Here is a: FACT. *If $S \overset{a.e}{\neq} T$, then $\exists E \in \mathcal{X}$ st. $d_E(S, T) > 0$.* This fact shows that m separates points in G, hence is a metric on G. (Optional: Prove the above FACT.)

b Fix sequences $\sigma_n \xrightarrow{n\to\infty} S$ and $\tau_n \xrightarrow{n\to\infty} T$ in (G, m). Our goal: THM. $\mathsf{m}\big(\sigma_n \tau_n, ST\big) \xrightarrow{n\to\infty} 0$.

First show that ISTFix a $B \in \vec{\mathbf{B}}$ and establish

2: $$d_B\big(\sigma_n \tau_n,\ ST\big) \xrightarrow{n\to\infty} 0\,.$$

Now prove (**??**) by using the triangle inequality and the above PROPOSITION.

c Produce an example of convergence

$$\tau_n \xrightarrow{n\to\infty} T \text{ in } (G, \mathsf{m}),$$

where each τ_n is invertible, but T is not. [*Hint:* This is the "creativity" part of the project. Necessarily, you can *not* have that $[\forall n, \ell: \tau_n \leftrightarrows \tau_\ell]$, since that would force the limit transformation to be invertible.]

Extra: Make each τ_n ergodic.

End of Home-C

C1:	___ ___	85pts
C2:	___ ___ ___	180pts
Total:	___ ___ ___	265pts

Please PRINT your Name

. . . .

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: . . . .