Calc 2
MAC2312

Home-C

Prof. JLF King
Touch: 4Oct2017

Hello. Take-home C is due, *slid under my office door*, by **6PM, Monday, 11 Dec., 1995**.

C1:

a Sum $\sum_{k=3}^{\infty} \frac{1}{k[k+3]} = \frac{U}{D}$, where $U=$ .... and $D=$ .... are co-prime posints.

[*Hint:* Telescoping series (several). "Co-prime" means "with no common factor".]

b Series $\sum_{n=2}^{\infty} \frac{[-1]^n}{n \cdot \log(n)}$ (*circle one*): **diverges, converges-conditionally, converges-absolutely**.

[*Hint:* Altenating-series-test and Integral-test.]

c Let $b_n := \frac{\log(n)+n}{1+2n}$. Then

$$\sum_{n=1}^{\infty} [b_n - b_{n+1}] = \text{..................................}.$$

[*Hint:* A real, $+\infty$, $-\infty$ or **DNEin**$\overline{\mathbb{R}}$.]

d Let $s_K := \sum_{n=3}^{K} \frac{[-1]^n}{\sqrt{n}}$ and let $L := \lim_{K\to\infty} s_K$.

What is the *smallest* value of K such that s_K approximates the limit L to within two decimal-places? [I.e, such that $|L - s_K| < \frac{1}{100}$.] Circle

$3 \leqslant K \leqslant 10$; $10 < K \leqslant 30$; $30 < K \leqslant 100$;
$100 < K \leqslant 1000$; $1000 < K \leqslant 10{,}000$; $10{,}000 < K \leqslant 10^5$;
$10^5 < K \leqslant 10^6$; $K > 10^6$.

e Let $f(x) := x^6 - 3x^4 + 2x + 1$. Compute the degree-3 Taylor polynomial of f, **centered at** -1.

TayPoly$(x)=$ ....

f For series $\sum_{n=3}^{\infty} \frac{[-2]^n}{3^{n+1}\, n^2} x^n$, its RoC= .... and its interval-of-convergence= ....

g Compute the real $\alpha=$ .... such that

$$3^\alpha \cdot \sum_{k=0}^{4000} \binom{4000}{k} 2^k \quad = \quad \sum_{j=0}^{1995} \binom{1995}{j} 8^j \,.$$

h,i [*Hint:* Theorem 9.25 might be useful.] Let's alter problem #**59**P*614*. Let N denote the *expected number* (average number) of rolls of the dice to first get a "2" or "11". (The original problem had "7" or "11".)

The series whose sum you want to compute is: ....

And its sum is $N=$ ....

*As essay problems, the original exam had two questions from the text; #**42**P634 and #**24**P630.*

C2: Let $G(x) := \mathsf{e}^{[x^2]} \cdot \int_0^x \mathsf{e}^{-[t^2]}\,dt$.

i Show that $G()$ is well-defined for *all* $x \in \mathbb{R}$; what theorems are you using to show this?

ii Show that $G'(x) = 1 + 2x \cdot G(x)$. What theorem(s) are you using?

iii For each $n \geqslant 1$, prove by induction that

$$G^{(n+1)}(x) \quad = \quad 2n \cdot G^{(n-1)}(x) + 2x \cdot G^{(n)}(x)\,.$$

iv Letting b_n be $G^{(n)}(0)/n!$, the the Maclaurin series for G is $\sum_{n=0}^{\infty} b_n x^n$. Derive an explicit formula for b_n in terms of factorials and powers.

v Compute (with proof) the radius of convergence, and the interval-of-convergence of the series.

[Parts (iv) and (v) are the most important. In (iv), your formula can have cases, e.g, "When n is a multiple of 3, then b_n equals ...".]

Notes: Good theorems to look up are: Alternating Series Thm; FTC; The Binomial Theorem (the elementary one, not the Binomial Series Theorem); IVT; MVT; The Ratio and Root Tests, and the various theorems of convergence of power series.

Recall that the interval-of-convergence of a power series can be open, half-open, or closed. Make sure to specify which endpoints are in your interval

For C1(a), play with telescoping series. For (c), recall that ***DNEinER*** means "does not exist in (the) extended reals", where the set of ***extended reals*** is the closed interval $[-\infty, +\infty]$. □

Filename: `Classwork/2Calculus/2Calc1995t/c-hm.2Calc1995t.latex`
As of: *Monday 31Aug2015.* Typeset: *4Oct2017 at 08:54.*