

ACT
MAA4211 7222

Home-C

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Touch: 17Nov2017

Hello. Essays violate the CHECKLIST at *Grade Peril!*
Exam is due by **3:30PM, Tuesday, 4Nov.2008**, slid completely under my office door, LIT402. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

Fill-in *all* blanks (*handwriting; don't bother to type*).

Definition. For a map $f: \Omega \rightarrow X$ between two MSes, let $\text{Cty}(f)$ denote f 's set of continuity-pts, and use $\text{DisCty}(f)$ for its set of discontinuity-pts. These sets are disjoint and their union is Ω . □

C1: Show no work.

a15 Define $X :=$ _____ $\subset \mathbb{R}$ st. the X -open ball $B := X\text{-Bal}_3(0) =$ _____ satisfies $B \subsetneq \text{Cl}_X(B) =$ _____ $\subsetneq X\text{-CldBal}_3(0) =$ _____

b Let $f: \mathbb{R} \rightarrow \mathbb{R}$ by $f(x) := [3x - x^3] - 1$. Define restrictions $g := f|_{[-2,1]}$ and $h := f|_{[-3,3]}$. Then the sup-norm $\|g\|_{\text{sup}} =$ _____ and $\|h\|_{\text{sup}} =$ _____.

c Using the stereographic-metric on \mathbb{R} : $\sigma\text{-Diam}(\text{Primes}) =$ _____.

d The limit in the seq #13P.62 is _____.

e Define $\mathbf{x} \in \mathbb{R}^{\mathbb{Z}^+}$ by $x_n := 1/3^n$. Each $p \in [1, \infty)$ has $\|\mathbf{x}\|_p =$ _____ . And $\|\mathbf{x}\|_p \rightarrow$ _____.

f Let $\varphi: \mathbb{R} \rightarrow \mathbb{R}$ by $\varphi(x) := \begin{cases} 2x & \text{if } x \text{ rational} \\ 1 - 3x & \text{if } x \text{ irrational} \end{cases}$. So $\text{Cty}(\varphi) =$ _____ and $\text{Range}(\varphi) =$ _____.

g30 Our MS is $X := \mathbb{R}_+ \times \mathbb{R}_+$ with the usual metric. Let $\Omega := \mathbb{Q}_+ \times \mathbb{Q}_+$. Define $f: X \rightarrow \mathbb{R}$ as: For $p \in X \setminus \Omega$, let $f(p) := 0$. For $p = (\frac{a}{q}, \frac{b}{r}) \in \Omega$, with a, b, q, r posints such that $a \perp q$ and $b \perp r$, let $f(p) := \frac{1}{q} - \frac{2}{r}$. So $\text{Cty}(f) =$ _____.

Essay questions: For each question, carefully type a triple-spaced essay solving the problem. Each essay starts a new page.

C2: A TS X is *countable self-dense* (CSD) if there exists a *countable* subset $D \subset X$ which is X -dense, i.e. $\text{Cl}_X(D) = X$.

i Prove that a seq-cpt **MS** (X, d) is necessarily CSD. You may use, without proof, that a countable union of countable sets is countable.

ii Produce a bounded and CSD MS (Y, m) , together with a seq $\vec{y} \subset Y$ with no Y -convergent subsequence.

C3: Produce (with proof) a sequence \vec{b} of non-negative reals such that $\sum_{n=0}^{\infty} b_n = +\infty$, and such that:

*Each monotone subsequence has **finite** sum.*

End of Home-C

C1: _____ 145pts

C2: _____ 85pts

C3: _____ 35pts

Poorly stapled, or missing ordinals : _____ -5pts

Missing names, or honor sigs : _____ -5pts

Total: _____ 265pts

HONOR CODE: "I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)." *Name/Signature/Ord*

Ord: _____
Ord: _____
Ord: _____