Linear Alg
MAS3114 2481

Class-C

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$$\mathsf{M} := \begin{bmatrix} & 2 & 7 & 2 & \\ & & 1 & 2 & 4 \\ 1 & & 1 & & \\ & 1 & 3 & & -2 \end{bmatrix}, \quad \mathsf{B} := \begin{bmatrix} -10 & -12 \\ 8 & 10 \end{bmatrix}.$$

Do not make approximations. Write expressions unambiguously e.g, "$1/a + b$" should be bracketed either $[1/a]+b$ or $1/[a+b]$. (Be careful with **negative** signs!) Question **C1** is short-answer. Questions **C2,C3** are essay questions.

C1: Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. (...or if the described matrix does not exist.)

z Prof. King sometimes gives freebie questions. Circle one: **True** **Yes** ***How would I know?***

y,x The above matrix B has distinct eigenvalues $\alpha < \beta$. Then α= ______ and β= ______. Compute an *integer matrix* $\mathsf{Q} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ so that $\mathsf{B} = \mathsf{Q}\left[\begin{smallmatrix} \alpha & 0 \\ 0 & \beta \end{smallmatrix}\right]\mathsf{Q}^{-1}$. Also, Q^{-1}= ______________.

a Rank(M)= ___. Det(M)= ___. Nullity(M)= ___.

b Give a basis for Nul(M) (as col-vecs)

Basis= $\{\qquad\qquad\}$

c Give a basis for ColSpn(M) (as col-vecs)

Basis= $\{\qquad\qquad\}$

d Give a basis for RowSpn(M) (as row-vecs)

Basis= $\{\qquad\qquad\}$

e What values of y put $\begin{bmatrix} 1 \\ y \\ 0 \\ 2 \end{bmatrix}$ in ColSpn(M)? [*Hint:* Either $\varnothing$, a single number, or $\mathbb{R}$.] ________________.

f Consider a 12×12 matrix $\mathsf{A}(y)$ populated with numbers, except that $\mathsf{A}_{2,6} = \mathsf{A}_{3,6} = \mathsf{A}_{9,6} = y$. I tell you that $\operatorname{Det}(\mathsf{A}(2)) = 4$ and $\operatorname{Det}(\mathsf{A}(3)) = 8$. Thus $\operatorname{Det}(\mathsf{A}(0)) =$ ______________________________.

g Let P be the 2×2 matrix realizing ortho-projection on the $y = -x$ line through the origin. Then

$$\mathsf{P} = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}.$$

h Permuting $(1 \ldots 6)$ to $(6,3,5,2,1,4)$ is Circle one: **Odd** **Even** **Neither**

i Let g be the summation poly of $f(t) := 2t^2 - t - 8$ in that $g(n) = \sum_{j=0}^{n-1} f(j)$, for each positive integer n. Then $g(n)$= _____ n^3+ _____ n^2+ _____ n+ _____.

Note. The following four questions ask for an example matrix which is neither the identity nor zero matrix. Give an example of...

j an idempotent matrix: ______________________.

k a nilpotent matrix: ______________________.

l an involution matrix: ______________________.

m a shear ______________ with determinant -1.

C2: On separate sheets of paper, give a careful statement of Cramer's Theorem, defining all the matrix-notation that you use.

For an 20×20 matrix E, let "$\operatorname{cof}(\mathsf{E}, i, j)$" be the 19×19 matrix which is E but with row i and col j deleted. Assume now that E is an invertible 20×20 matrix. Let e be the $(3,7)$-entry in E^{-1}. Give a formula, ITOf cofactors of E and determinants, for $e =$ ______________________________________.

C3: *On separate sheets of paper, show all work for this problem, writing in complete sentences that end with visible periods.* :-) Consider the unit vector $\mathbf{u} = \frac{1}{5}(0,3,4)$. Compute a 2-decimal approximation to the

3×3 matrix R which rotates CCW about **u** by 30°. So

$$\mathsf{R} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}.$$

End of Class-C

C1:	___ ___ ___	420pts
C2:	___ ___	60pts
C3:	___ ___	45pts
Total:	___ ___ ___	525pts

Please PRINT your name and ordinal. Ta:

Ord:

. . . .

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature:

. . . .