Topo 1: MTG4302 and MTG5316 — **Class-C** — Prof. JLF King, *Touch*: 16Aug2016

"*Finishing the Semester in Style*"

Exam date **27Oct1997**.

Abbreviations. TS, MS, topological/metric space. seq-cpt, sequentially-compact. For a TS Y, we use $\overline{A}$ for the closure of A, and $A^c = Y \setminus A$ for the complement of A.

A TS is ***regular*** if singletons are closed and: *For each closed set C and point $p \notin C$, there exist disjoint open sets P and U such that $P \ni p$ and $U \supset C$.*

A TS is ***normal*** if singletons are closed and: *For each pair of disjoint closed sets C_1, C_2, there exist disjoint open sets U_j st. $U_j \supset C_j$.*

C1: Here Y is a topological space.
Y is ***Hausdorff*** if...
Y is ***compact*** if...
For $(b_n)_{n=1}^{\infty}$ a sequence, $b_n \to c$ means that...
Our Y is ***sequentially-compact*** if ...
Fix a MS (M, d). A subset $R \subset M$ is ***ε-separated*** if... A subset $N \subset M$ is ***ε-spanning*** if... The MS M is ***totally-bounded*** if...

C2: Give examples, with proof, of the following:

a) A T_1-space X which is not Hausdorff.

b) A Hausdorff space Y which is not regular.

c) A topology $\mathcal{T}$ on $\mathbb{R}$ –<u>not</u> the discrete topology– so that: *The $\mathcal{T}$-convergent sequences are precisely the eventually-constant sequences.*

C3: Give an example (with proof) of a product, Λ, of seq-cpt spaces, which is not seq-cpt.

C4: Consider a MS (X, d) and an open cover $\mathcal{C}$ of X.

i) Define what it means for a particular positive real δ to be a ***Lebesgue number*** of $\mathcal{C}$.

ii) Suppose now that X is sequentially-compact. Prove that $\mathcal{C}$ has a Lebesgue number.

C5: Suppose that $(Z, \mathcal{T})$ is a compact Hausdorff space. Prove that Z is normal.

C6: Putting the usual metric on $[0, 1/n]$, let

$$\Omega := \prod_{n=1}^{\infty} [0, \tfrac{1}{n}]$$

—<u>*but*</u> equipped with the Box topology, $\mathcal{B}$.

a) Prove or disprove: $(\Omega, \mathcal{B})$ is sequentially-compact.

b) Prove that $(\Omega, \mathcal{B})$ is *not* LCG (locally countably generated), hence not metrizable.

Bonus: Produce, with proof, an open cover $\mathcal{C}$ of $\mathbb{R}$ (usual topology), which has no Lebesgue number.

End of Class-C