Staple!

Ord: ___ ___

Sets and Logic MHF3202 17HE | **Class-C** | Prof. JLF King Monday, 04Apr2022

C1: Short answer. Show no work.
Write **DNE** if the object does not exist or the operation cannot be performed. NB: DNE $\neq \{\} \neq 0$.

___ a For a LOR (letter-of-recommendation), Prof. K requires two courses, or a Special Topics or graduate course [Circle]:

Yes **True** **Darn tootin'!**

___ b Between sets $\mathbf{X} := \mathbb{Z}_+$ and $\mathbf{\Omega} := \mathbb{N}$, consider injections $g{:}\mathbf{X}\hookrightarrow\mathbf{\Omega}$ and $h{:}\mathbf{\Omega}\hookrightarrow\mathbf{X}$, defined by

$$g(x) := 3x \quad \text{and} \quad h(y) := y+5\,.$$

Schröder-Bernstein produces a set $B \subset h(\mathbf{\Omega}) \subset \mathbf{X}$ st., letting $F := \mathbf{X} \smallsetminus B$, the fnc $\theta{:}\mathbf{X}\hookrightarrow\!\!\!\!\rightarrow\mathbf{\Omega}$ is a *bijection*, where

$$*: \qquad \theta\lfloor_F := g\lfloor_F \quad \textit{and} \quad \theta\lfloor_B := h^{-1}\rfloor_B\,.$$

For this (g,h), the (F,B) pair is unique. Computing, $\theta(56)=$ ⌊ . . . . . ⌋. $\theta(137)=$ ⌊ . . . . . ⌋. $\theta^{-1}(603)=$ ⌊ . . . . . ⌋.

___ c Let $\mathbb{S}_\mathbb{N}$ be the set of *permutations of* $\mathbb{N}$. [Circle] those of following sets which are equinumerous with $\mathbb{N}^\mathbb{N}$:

$\mathbb{N}$ $\mathbb{R}$ $\mathbb{N}\times\mathbb{R}$ $2^\mathbb{R}$ $\mathbb{R}^\mathbb{N}$ $\mathbb{R}^\mathbb{R}$ $\mathbb{S}_\mathbb{N}$

[Possibly none or several. Schröder-Bernstein may be useful.]

___ d Euler $\varphi(121000) =$ ⌊ . . . . . ⌋.
Express your answer as a product ${p_1}^{e_1}\cdot{p_2}^{e_2}\cdot\ldots$ of *primes* to posint powers, with $p_1 < p_2 < \ldots$.

___ e Number $N := [45]^6 = p^{\text{Power}}\cdot q^{\text{Power}}$ for distinct primes $p=$ ⌊ . . . . . ⌋ and $q=$ ⌊ . . . . . ⌋. The *divisor-sum* is $\sigma_1(N) = A{\cdot}B$ where $A=$ ⌊ . . . . . ⌋ & $B=$ ⌊ . . . . . ⌋ are fractions related to p and q, respectively.
Similarly, $\sigma_2(N) =$ ⌊ . . . . . ⌋ · ⌊ . . . . . ⌋.

OYOP: *In grammatical English* ***sentences****, write your essays on every* 2^{nd} *line* (usually)*, so I can easily write between the lines.*

C2: Let $\mathbf{J} := [3,7]$, an interval of reals. You may use, without proof, the Schröder-Bernstein thm and:

$\mathbf{a_1}$: $\mathbb{R} \asymp \{0,1\}^\mathbb{N}$. **$\mathbf{a_2}$:** $\mathbb{N}\times\mathbb{R} \asymp \mathbb{R}$.

$\mathbf{a_3}$: *For each three sets* Ω,B,D: $\Omega^{B\times D} \asymp \left[\Omega^B\right]^D$.

$\mathbf{a_4}$: *The set* $S := \mathbb{Q}\cap\mathbf{J}$ *is denumerable.*

Prove that $\mathbf{C}(\mathbf{J})$*, the set of continuous functions* $\mathbf{J}\to\mathbb{R}$*, is bijective with* $\mathbb{R}$. Cite each ($\mathbf{a}_i$) where you use it. Specify what Ω, B, D are, when you apply ($\mathbf{a_3}$). [*Note:* Split your proof into easily-stated lemmas.]

C1:	___ ___ ___	160pts
C2:	___ ___	55pts
Total:	___ ___ ___	215pts

NAME: ⌊ . . . . . ⌋

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor or TA."*

Signature: ⌊ . . . . . ⌋