

C1: Short answer. Show no work.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\}$ $\neq 0$.

z See Canvas, (C1z).

a Sum $\binom{45}{19} + 2\binom{45}{20} + \binom{45}{21} = \binom{T}{B}$, where $T =$ _____ and $B =$ _____. Using the same idea,

$$\binom{32}{13} + 3\binom{32}{14} + 3\binom{32}{15} + \binom{32}{16} = \binom{\tau}{\beta},$$

where $\tau =$ _____ and $\beta =$ _____.

b Relation **R** is a binrel on set \mathbb{Z}_+ , defined by xRy IFF $x^2 = 5y$.

Assertion “Relation **R** is reflexive” is T F
 Assertion “Relation **R** is antireflexive” is T F

c On $\Omega := [1..29] \times [1..29]$, define binary-relation **C** by: $(x, \alpha) C (y, \beta)$ IFF $x \cdot \beta \equiv_{30} y \cdot \alpha$. Statement “Relation **C** is an **equivalence relation**” is: T F

d For a finite list \mathcal{S} of posints, define

$$\mu_{\mathcal{S}}(N) := \left\{ k \in [1..N] \mid \exists d \in \mathcal{S} \text{ with } d \bullet k \right\}.$$

Use Inclusion-Exclusion to compute

$$|\mu_{\{6,7,9\}}(67)| = \text{_____}$$

Essay. Hand-write your essay on paper, writing large and dark. [You may typeset it if you wish, but that is likely slower.] *Do Not Scrunch!* You can write on every 2nd-line to make your essay easier to read. The essay is written in complete sentences, correctly spelled and punctuated, and assembled into logical paragraphs.

When the essay is done, convert it to a pdf, then name it as follows: **C2.<your name>.pdf**

E.g, Rachel Stein will name her pdf file as

C2.Rachel-Stein.pdf

There are no spaces in the filename! The extension is “pdf”. A hyphen is used to separate the given-name from the family-name. (You may write names in the order you are accustomed.) \square

52 C2: Define a sequence $\vec{b} = (b_0, b_1, b_2, \dots)$ by $b_0 := 0$ and $b_1 := 5$ and

$$\dagger: \quad b_{n+2} := 11b_{n+1} - 24b_n, \quad \text{for } n = 0, 1, \dots$$

Use induction to prove, for each natnum k , that

$$\ddagger: \quad b_k = 8^k - 3^k.$$

8 C3: “I have neither requested nor received help on this exam other than from my professor.”

C1: _____ 100pts

C2: _____ 52pts

C3: _____ 8pts

Total: _____ 160pts