Sets and Logic
MHF3202 2787

IndividualOP-C

Prof. JLF King
Wednesday, 24Oct2018

C1: Short answer. Show no work.

a A *minimum* requirement for an LOR (letter-of-recommendation) from Prof. K is two courses. Circle:

Yes **True** **Darn tootin'!**

b The basic theory of infinite cardinals was developed by Circle: **Abel** **Alladi** **Avogadro** **Bernstein** **Bertrand** **Cantor** **Cauchy** **Dedekind** **Fraenkel** **Gauss** **Hilbert** **Russell** **Shakespeare** **Zermelo**

He started this work in latter half of the Circle:

1400s **1500s** **1600s** **1700s** **1800s** **1900s**

c For $G := \{1, 2, 3, 4\}$, consider $f : G \to \mathcal{P}(G)$ by

$$f(1) := G, \qquad f(2) := \{1, 3\},$$
$$f(3) := \varnothing, \qquad f(4) := \{1, 4\}.$$

The set $B := \{x \in G \mid f(x) \not\ni x\}$ is $\{$ .... $\}$.

d The map $f(k, n) := 2^k \cdot [1 + 2n]$ is a bijection from $\mathbb{N}\times\mathbb{N} \hookrightarrow\!\!\!\!\rightarrow \mathbb{Z}_+$. And $f^{-1}(176) = ($ .... , ... $)$.

e A "Cantor's-Hotel" type bijection $f : (5, 6] \hookrightarrow\!\!\!\!\rightarrow (0, 1)$ is:

$f($ .... $) :=$ .... , for *each* posint n;

and $f(x) :=$ .... , for *each* $x \in (5, 6] \smallsetminus C$,

where $C :=$ ....

f Let $\mathcal{P}_\infty$ denote the family of all *infinite* subsets of $\mathbb{N}$. Define relation $\approx$ on $\mathcal{P}_\infty$ by: $A \approx B$ IFF $A \cap B$ is infinite. Stmt "*This $\approx$ is an equivalence-relation*" is: *T* *F* g Define G:[1 .. 12]↺ where $G(n)$ is the number of letters in the n^{th} Gregorian month. So $G(2) = 8$, since the 2nd month is "February". The only fixed-point of G is ________. The *set* of posints k where $G^{\circ k}(12) = G^{\circ k}(7)$ is ____.

[January, February, March, April, May, June, July, August, September, October, November, December]

C2: Short answer. Show no work.

Between sets $\mathbf{X} := \mathbb{Z}_+$ and $\mathbf{Y} := \mathbb{N}$, consider injections $f{:}\,\mathbf{X}\hookrightarrow\mathbf{Y}$ and $h{:}\,\mathbf{Y}\hookrightarrow\mathbf{X}$, defined by

$$f(x) \;:=\; 3x \quad\text{and}\quad h(y) \;:=\; y+5\,.$$

Schröder-Bernstein produces a set $G \subset h(\mathbf{Y}) \subset \mathbf{X}$ st., letting $U := \mathbf{X} \setminus G$, the fnc $\beta{:}\,\mathbf{X}\hookrightarrow\!\!\!\!\rightarrow\mathbf{Y}$ is a *bijection*, where

$*$: $$\beta\rfloor_U \;:=\; f\rfloor_U \quad\text{and}\quad \beta\rfloor_G \;:=\; h^{-1}\rfloor_G\,.$$

For <u>this</u> (f,h), the (U,G) pair is unique. Computing,
$\beta(56)=$ .... $\beta(137)=$ .... $\beta^{-1}(603)=$ ....

C1:	___ ___ ___	145pts
C2:	___ ___	50pts
Total:	___ ___ ___	195pts