

**Abbrevs.** Let **SCC** mean “positively oriented simple-closed-contour”. For a **SCC**  $C$ , have  $\dot{C}$  be the (open) region  $C$  encloses, and let  $\hat{C}$  mean  $C$  together with  $\dot{C}$ . So  $\hat{C}$  is  $C \cup \dot{C}$ ; it is automatically simply-connected and is a closed bounded set.

**C1:** Short answer. Show no work.  
Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

**z** The person who completed the proof that  $\oint_C f = 0$ , when  $f$  holomorphic on  $\hat{C}$ , was: **Archimedes Argand Abel Euler Gauss Goursat Hadamard Hypatia Lagrange Liouville Morera Picard Rouché Taylor Weierstrass Zeno**

**a** Compute  $\int_0^{2\pi} \frac{1}{\cos(\theta) + 6} d\theta =$  .....  
[Hint: CoV  $z = e^{i\theta}$ .]

**b**  $\int_{-\infty}^{\infty} \frac{1}{[x^2 + 1]^2} dx =$  .....  
[Hint: A  $\square$  contour.]

**c**  $\int_{-\infty}^{\infty} \frac{\cos(x)}{x^2 + 9} dx =$  .....  
[Hint: A  $\square$  contour. Sine is an odd fnc, so  $\int_{-\infty}^{\infty} \frac{\sin(x)}{x^2 + 9} dx$  is zero.]

**d**  $\text{Res}\left(\frac{e^{2z}}{[z - 5]^4}, z=5\right) =$  .....

**e**  $\text{Res}\left(z^4 \cdot e^{1/z}, z=0\right) =$  .....

OYOP: In grammatical English *sentences*, write your essay on every *third* line (usually), so that I can easily write between the lines.

**C2:** **i** Carefully state, but DNP [Do Not Prove] Cauchy's Inequality theorem.

**ii** Define: A function  $f$  is *entire* if...  
Carefully state, but DNP, Liouville's thm on entire fncs.

**iii** State the Gauss Mean Value Thm.

**iv** Derive Gauss-MVT directly from the Cauchy Integral Formula.

End of Class-C

**C1:**    \_\_\_ \_\_\_    175pts

**C2:**    \_\_\_ \_\_\_    90pts

**Total:**    \_\_\_ \_\_\_    265pts