Linear Algebra
MAS3114 3164

Class-C

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Hello. Please write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797···.

C1: Show no work.

a) The seq. $\vec{\mathbf{g}} := (g_n)_{n=-\infty}^{\infty}$ is defined by recurrence

$$g_{n+2} = 3g_{n+1} + -1g_n$$

and initial conditions $g_0 := -1$ and $g_1 := 2$. So its n^{th} term is $g_n = C_1 \cdot \mu^n + C_2 \cdot \nu^n$, where $\mu<\nu$ are real, and

$C_1=$ ________, $\mu=$ ________,

$C_2=$ ________ and $\nu=$ ________.

[*Hint:* The corresponding matrix is $\mathsf{G} := \left[\begin{smallmatrix}3 & -1\\ 1 & 0\end{smallmatrix}\right]$. And μ, ν are its eigenvalues.]

b) $\mathsf{M} := \begin{bmatrix}-7 & 9 & 0\\ -6 & 8 & 0\\ 12 & -18 & -1\end{bmatrix}$ has three real eigenvalues,

$\alpha=$ ________ $\leqslant \beta=$ ________ $\leqslant \gamma=$ ________.

Hence $\left[\begin{smallmatrix}\alpha & 0 & 0\\ 0 & \beta & 0\\ 0 & 0 & \gamma\end{smallmatrix}\right] = \mathsf{U}^{-1}\mathsf{M}\mathsf{U}$, where

$$\mathsf{U} = \left[\quad\middle|\quad\middle|\quad\right].$$

c) $\mu=$ ________ $\leqslant \nu =$ ________

are the eigenvalues of $\mathsf{G} := \begin{bmatrix}3 & 1\\ 5 & 2\end{bmatrix}$. Let $\mathsf{D} := \begin{bmatrix}\mu & 0\\ 0 & \nu\end{bmatrix}$. Then $\mathsf{D} = \mathsf{U}^{-1}\mathsf{G}\mathsf{U}$ where the 2×2 <u>*integer*</u> matrix U is

$$\mathsf{U} = \left[\quad\right].$$

d) The 3×3 elem-matrix whose lefthand action adds 8 times row-2 to row-1 is $\left[\quad\right]$.

e) Suppose T is a linear map from $\mathbb{R}^3$ to $\mathbb{R}^2$. Let $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the standard basis for $\mathbb{R}^3$, and let $\{\mathbf{v}_1, \mathbf{v}_2\}$ be the standard basis for $\mathbb{R}^2$. Suppose that $T(\mathbf{e}_1) = 17\mathbf{v}_1 - 2\mathbf{v}_2$ and $T(\mathbf{e}_2) = 6\mathbf{v}_2$ and $T(\mathbf{e}_3) = -4\mathbf{v}_1 - 3\mathbf{v}_2$.

Then the matrix of T is: ________.

f) Let $\mathbf{v}_1 := \left[\begin{smallmatrix}0\\ 1\\ -1\\ 0\end{smallmatrix}\right]$, $\mathbf{v}_2 := \left[\begin{smallmatrix}2\\ 1\\ -2\\ 3\end{smallmatrix}\right]$, $\mathbf{v}_3 := \left[\begin{smallmatrix}4\\ W\\ 3\\ Y\end{smallmatrix}\right]$, So $\mathbf{v}_3 \in \operatorname{Spn}(\mathbf{v}_1, \mathbf{v}_2)$ when $W=$ ________ & $Y=$ ________.

And $\mathbf{v}_3 = \alpha\mathbf{v}_1 + \beta\mathbf{v}_2$, where $\alpha=$ ________ and $\beta=$ ________.

Essay question: On your own sheets of paper, write a soln using complete sentences, explaining a bit about HOW this problem is solved.

C2: Let $\mathsf{B} := \begin{bmatrix}0 & 7 & 3\\ 1 & 0 & -1\\ 0 & 2 & 1\end{bmatrix}$. Please write its inverse matrix as a product of elementary matrices. Please write, <u>below</u> each matrix, the corresponding row-operation symbol: $\mathbb{S}_{i,j}$ (switch two rows), $\mathbb{M}_{i,\beta}$ (mult. Row$_i$ by β), $\mathbb{A}_{i,(\alpha,j)}$ (to Row$_i$ add $\alpha \cdot$ Row$_j$).

$\mathsf{B}^{-1}=$ ________.

C3: The fol. 2×2 matrix does <u>**not**</u> have...

$$\mathsf{B} := \left[\quad\middle|\quad\right]$$

...an eigenbasis, because the algebraic multiplicity of e-val $\mu=$ ____ strictly exceeds its geometric multiplicity. Show me this by rref-ing the matrix $\mathsf{B} - \mu\mathbf{I}$ and showing that it has too few free-cols.

C1:	___ ___ ___	190pts
C2:	___ ___	45pts
C3:	___ ___	50pts
Total:	___ ___ ___	285pts

Print name ________ Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ________