Linear Algebra | **Class-C** | Prof. JLF King
MAS4105 *6137* | | Tuesday, 17Nov2015

C1: Show no work. Please write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

___ ___ [z] A multivariate polynomial, where each monomial has the same degree, is (circle)

monogamous **atrocious** **gregarious**
monic **expialadocious** **homogeneous**
manic **unitary** **Unitarian** **utilitarian**

___ ___ [a] Suppose C and A are 3×3 matrices s.t $\operatorname{Det}(\mathsf{C}) = \frac{1}{2}$ and $\operatorname{Det}(\mathsf{A}) = 5$. Then

$\operatorname{Det}\big(\mathsf{C}^{-1}\mathsf{A}\mathsf{C}^{\mathsf{t}}\,\mathsf{A}^{\mathsf{t}}\,\mathsf{A}\mathsf{C}^{\mathsf{t}}\big) =$ ....

___ ___ [b] Let $\mathbb{S}_8$ denote the set of permutations of $[1\,..\,8]$. For an 8×8 matrix $\mathsf{M} = [\beta_{i,j}]_{i,j}$, write the *"Generalized-diagonal formula"*

$$\operatorname{Det}(\mathsf{M}) = \sum_{\text{......}} \Big[\text{................................} \Big].$$

___ ___ [c] $\mathsf{M} := \begin{bmatrix} 70 & 7 \\ 1 & 2 \end{bmatrix}$. Compute M^{-1} over these three fields. [Write your $\mathbb{Z}_p$ answers using symmetric residues.]

Over $\mathbb{Z}_5$: $\mathsf{M}^{-1} =$ .... Over $\mathbb{Z}_7$: $\mathsf{M}^{-1} =$ ....

Over $\mathbb{Q}$: $\mathsf{M}^{-1} =$ ....

___ ___ [d] $\mu =$ .... $\leqslant \nu =$ ....

are the eigenvals of $\mathsf{G} := \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$. Let $\mathsf{D} := \begin{bmatrix} \mu & 0 \\ 0 & \nu \end{bmatrix}$. Then $\mathsf{D} = \mathsf{U}^{-1}\mathsf{G}\mathsf{U}$ where the 2×2 <u>*integer*</u> matrix U is

$$\mathsf{U} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}.$$

OYOP: Essays: *Write on every* ***third*** *line, so that I can easily write between the lines. In grammatical English* ***sentences****, prove the following:*

C2: DEFN: A collection $\mathcal{C} := \{\mathbf{U}_1, \ldots, \mathbf{U}_K\}$ of <u>subspaces</u> is ***linearly-independent*** if: . . .

THM: For linear-transformation $\mathsf{T}: \mathbf{V} \to \mathbf{V}$, eigenspaces $\mathbf{W}_1, \ldots, \mathbf{W}_8$ have (distinct) eigenvalues $\beta_1, \ldots, \beta_8$. Prove that $\mathcal{D} := \{\mathbf{W}_1, \ldots, \mathbf{W}_8\}$ is linearly-independent.

C3: Matrix $\mathsf{M} = \left[\begin{smallmatrix} \mathsf{A} & \mathsf{B} \\ \mathsf{C} & \mathsf{D} \end{smallmatrix}\right]$, where A and D are 5×5 and 7×7, resp. Suppose C is the 7×5 zero-matrix. <u>Prove</u> that $\operatorname{Det}(\mathsf{M}) = \operatorname{Det}(\mathsf{A}) \cdot \operatorname{Det}(\mathsf{D})$. [*Hint:* A good picture helps.]

End of Class-C

C1:	___ ___ ___	130pts
C2:	___ ___	55pts
C3:	___ ___	55pts
Total:	___ ___ ___	240pts