Linear Algebra MAS4105 *6137*	**Class-C**	Prof. JLF King *Touch*: 17Nov2015

OYOP: For the 2 Essays: *Write your grammatical English* ***sentences*** *on every* ***third*** *line, so that I can easily write between the lines.*

C1: DEFN: A collection $\mathcal{C} := \{\mathbf{W}_1, \ldots, \mathbf{W}_8\}$ of <u>subspaces</u> is ***linearly-independent*** if: **. . .**

THM: For linear-transformation $\mathsf{T}: \mathbf{V} \to \mathbf{V}$, eigenspaces $\mathbf{W}_1, \ldots, \mathbf{W}_8$ have (distinct) eigenvalues $\beta_1, \ldots, \beta_8$. Prove that $\mathcal{D} := \{\mathbf{W}_1, \ldots, \mathbf{W}_8\}$ is linearly-independent.

C2: Matrix $\mathsf{M} = \left[\begin{smallmatrix} \mathsf{A} & \mathsf{B} \\ \mathsf{Z} & \mathsf{D} \end{smallmatrix}\right]$, where A and D are 5×5 and 7×7, resp., and Z is 7×5. <u>Prove</u> that if a GD (generalized diagonal) passes through the B block, then it passes through Z.

Short answer, OYOP:

C3: A system of 3 linear equations in unknowns $x_1, \ldots, x_5$ reduces to the augmented matrix

$$\begin{bmatrix} 5 & 4 & 0 & 0 & -9 & | & -7 \\ 0 & 0 & 3 & 0 & 8 & | & -3 \\ 0 & 0 & 0 & 2 & 1 & | & 0 \end{bmatrix},$$

which is in RREF. Please circle each *pivot entry.*

OYOP, describe the *general solution* in this form,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \alpha \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \beta \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \gamma \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix} + \ldots$$

where each $\alpha, \beta, \gamma, \delta, \ldots$ is a free variable (either x_1 or…or x_5), and each column vector has specific numbers in it. $\text{Dim}(\textit{SolnFlat}) =$ ________________ .

C4: Show no work.

Please write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

a Let $\mathsf{M}(x) := \begin{bmatrix} 3x-2 & 7x^4-8 & 10 \\ 5 & 9x-2 & 2x-8 \\ 8x & x^5-2 & x^3+2 \end{bmatrix}$.

The high-order term of polynomial $\text{Det}(\mathsf{M}(x))$ is Cx^N, where $C =$ ________________ and $N =$ ________________ .

b Let $\mathbb{S}_8$ denote the set of permutations of $[1\,..\,8]$. For an 8×8 matrix $\mathsf{M} = (\beta_{i,j})_{i,j}$, write the *"Generalized-diagonal formula"*

$\text{Det}(\mathsf{M}) =$ ________________________________ .

c Let $\mathbf{v}_1 := \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 := \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$, $\mathbf{v}_3 := \begin{bmatrix} 4 \\ Y \\ 3 \end{bmatrix}$. Our $\mathbf{v}_3$ is in $\text{Spn}(\mathbf{v}_1, \mathbf{v}_2)$ when number $Y =$ ________ . And then, $\mathbf{v}_3 = \alpha\mathbf{v}_1 + \beta\mathbf{v}_2$, where $\alpha =$ ____________ and $\beta =$ ____________ .

d The seq. $\vec{\mathbf{g}} := (g_n)_{n=-\infty}^{\infty}$ is defined by recurrence

$$g_{n+2} = 3g_{n+1} + 4g_n$$

and initial conditions $g_0 := -1$ and $g_1 := 11$. So its n^{th} term is $g_n = C_1 \cdot \mu^n + C_2 \cdot \nu^n$, where $\mu < \nu$ are real, and

$C_1 =$ ____________________ , $\mu =$ ____________________ ,

$C_2 =$ ____________________ and $\nu =$ ____________________ .

[*Hint:* The corresponding matrix is $\mathsf{G} := \left[\begin{smallmatrix} 3 & 4 \\ 1 & 0 \end{smallmatrix}\right]$. And μ, ν are its eigenvalues.]