Linear Algebra
MAS4105 *3247*

Class-C

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C1: With $\mathbf{V}$ a vectorspace over field $\mathbf{F}$, suppose $\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_3, \ldots \subset \mathbf{V}$ are subspaces. Define sets

$$\mathbf{X} := \mathbf{W}_1 \cup \mathbf{W}_2 \,; \quad \mathbf{S} := \left\{ \mathbf{u} + 2\mathbf{v} \;\middle|\; \begin{matrix} \mathbf{u} \in \mathbf{W}_1 \text{ and} \\ \mathbf{v} \in \mathbf{W}_2 \end{matrix} \right\};$$
$$\mathbf{Y} := \bigcap_{n=1}^{\infty} \mathbf{W}_n \,.$$

OYOSOP, *prove*, or give an *explicit CEX* (field, VS and vectors/scalars) to: "*Set* $\mathbf{X}$ *is a VS.*" Ditto $\mathbf{S}$ and $\mathbf{Y}$.

C2: Matrix R equals $\mathit{RREF}(\mathsf{S})$, where S is

$$\begin{bmatrix} 0 & 1 & 2 & 0 & 0 & 2 & 1 \\ 0 & 1 & 2 & 1 & -1 & 3 & -1 \\ 0 & -1 & -2 & -1 & 2 & -3 & 3 \\ 0 & 2 & 4 & 1 & -1 & 5 & 0 \end{bmatrix} \text{ and } \mathsf{R} := \begin{bmatrix} 0 & 1 & 2 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

So Rank(S)= ____________ and Nullity(S)= ____________.

OYOSOP, as col-vecs, write a basis $\mathcal{A}$ for Range(L_{S}). As column-vectors, write a basis $\mathcal{B}$ for Ker(L_{S}).

C3: a On $\mathbf{V} := \mathrm{MAT}_{5\times3}(\mathbb{Q})$, the map $\mathsf{T}\colon \mathbf{V} \to \mathbf{V}$ by $\mathsf{T}(\mathsf{A}) := \mathit{RREF}(\mathsf{A})$ is $\mathbb{Q}$-linear. *T* *F*

The map $\mathrm{PLY}_3 \to \mathrm{PLY}_3$ which sends $f \mapsto g$, where $g(x) := x \cdot f'(x+5)$, is: Circle best Linear Affine Neither

___ ___ b Let R_θ be the std. rotation [by θ] matrix. With

$$\mathsf{E} := \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix} \quad \text{and} \quad \mathsf{D} := \begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix},$$

the product $[\mathsf{ED}]^{23} = \alpha \cdot \mathsf{R}_\theta$, with $\alpha=$ ______________ $\in \mathbb{R}$ and $\theta=$ ______________ $\in (-180°, 180°]$. [*Hint:* Don't multiply matrices. Geometrically, E and D represent what lin-trns?]

___ ___ c With C the change-of-basis matrix from $\mathcal{E} := (1, x, x^2)$ to $\mathcal{B} := (3x + 5x^2, x + 2x^2, 1)$, then C^{-1} equals

$$\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}, \ \mathsf{C} = \begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}.$$

___ ___ d The determinant of $\mathsf{M} := \begin{bmatrix} 3 & 4 & 5 \\ 0 & 5 & -1 \\ 0 & 7 & -2 \end{bmatrix}$ is ______. The char-poly of M is $Ax^3 + Bx^2 + Cx + D$, where $B=$ ______ and $C=$ ______.

End of Class-C

C1:	___ ___	80pts
C2:	___ ___	65pts
C3:	___ ___ ___	130pts
Total:	___ ___ ___	275pts

Please PRINT your name and ordinal. Ta:

Ord:

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: