Differential Eqns MAP2302 | **C-Class** | Prof. JLF King *Touch*: 16Nov2015

I give permission for Prof. King to email my grades to my `ufl.edu` *eddress.* Circle: Yes No

Notation. For the *Laplace transform* of f, use $\mathcal{L}(f) = \widehat{f}$. Use $\mathcal{L}^{-1}$ for the inverse Laplace-transform operator.

C1: Show no work.

___ ___ **a** If $\lim_{x\to 0+} 8/x$ equals ∞, then $\lim_{x\to 0+} 5/x$ is Circle:

Prof. King's beret. *Colored chalk.*

___ ___ **b** Fnc $y_{\alpha,\beta}(t) = \alpha e^{At} + \beta e^{Bt} + P\cdot\sin(t) + Q\cdot\cos(t)$ is the general soln to

$$*: \quad 3y'' + 4y' + y = \cos(t),$$

with numbers $A=$ ____ , $B=$ ____ , $P=$ ____ , $Q=$ ____ .

Also, the *constants* on LhS($*$) are $3, 4, 1$. With the DE describing the position of a spring, the *constant* corresponding to Hooke's constant is ____ .

___ ___ **c** Fncs $x(t)$ and $y(t)$ satisfy this system of DEs,

$$x' + 4x - y = 0,$$
$$y' + 2x + 7y = 0.$$

It can be written $Y' = G\cdot Y$, where $Y := \left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ and G is matrix ____ .

Characteristic poly of G is $\wp_G(z)=$ ____

A soln has $x(t)$ a linear combination of $e^{\alpha t}$ and $e^{\beta t}$ for *numbers* $\alpha=$ ____ and $\beta=$ ____ .

___ ___ **d** Matrices G, A, P are 3×3, with G invertible and P nilpotent.

Matrix GPG^{-1} *is nilpotent*: *AT AF Nei*

Each entry of e^{tP} *is a polynomial*: *AT AF Nei*

Matrix e^{P} *is nilpotent*: *AT AF Nei*

P^2 *is the zero-matrix*: *AT AF Nei*

The mult-inverse of e^{G} *is* $-e^{G}$: *AT AF Nei*

Matrix $e^{[A^2]}$ *equals* $[e^{A}]^2$: *AT AF Nei*

e U.F. $x = x(t)$ satisfies $2x^{(3)} + 5x^{(2)} - x = 0$.

Then $Y := \begin{bmatrix} x \\ x' \\ x'' \end{bmatrix}$ satisfies $Y' = M\cdot Y$, where M is this 3×3 matrix of numbers:

$$M = \begin{bmatrix} \; & \; & \; \\ \; & \; & \; \\ \; & \; & \; \end{bmatrix}.$$

f We can re-write function

$$f(t) := 2\cdot\cos\left(\tfrac{11}{6}\pi + 4t\right) + \sqrt{3}\cdot\cos(\pi + 4t)$$

as $f(t) = R\cdot\cos(\theta + 4t)$, for **real** numbers $R=$ ____ $\geqslant 0$ and $\theta=$ ____ $\in [0, 2\pi)$.

[*Hint:* OYOP, write cos() as the real-part of exp(*something*), and *Draw Yourself a large Useful Picture* in the complex plane.]

g Suppose $y(0) = -1$, $y'(0) = 5$, $y''(0) = 2$. Then $\mathcal{L}\left(y^{(3)} + y^{(2)} - 4y\right)(s)$ equals $\left[[B(s)\cdot\widehat{y}(s)] + C(s)\right]$ for **polynomials**

$C(s)=$ ____

and $B(s)=$ ____ .

End of C-Class

C1: ___ ___ ___ 185pts

Total: ___ ___ ___ 185pts