NT Summer
MAS4203 4801

Class-C

Prof. JLF King
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C1: *Show no work.*

a The author of our text is Circle: **DNE Euler Fermat *Waldo* Pollard Fuchs Gauss Archimedes Silverman Sanders Shannon Strayer Tony Tori**

b The smallest natnum which is *not* a sum of three squares is ⌊ . . . . . ⌋.

c A primitive Pythagorean triple has $28^2 + b^2 = c^2$, where $b =$ ⌊ . . . . . ⌋ and $c =$ ⌊ . . . . . ⌋ with $b \perp c$.

d Let $g := \boldsymbol{\sigma}^{\circledast -1}$ [i.e, the convol-inverse of the divisor-sum fnc]. So $g(2) =$ ⌊ . . . . . ⌋, $g(9) =$ ⌊ . . . . . ⌋ and $g(18) =$ ⌊ . . . . . ⌋

e Let $f(x) := x^2 - 4x - 2$, and $Z_1 := t_0 := 3$. Note $f(Z_1) \equiv_5 0$. Note $f'(Z_1) =$ ⌊ . . . . . ⌋ $\not\equiv_5 0$. Use Hensel's lemma to compute coefficients $t_k \in [0\,..\,5)$ [*put them in the blanks, below*]

$$Z_4 = \overbrace{\underbrace{t_0 \cdot 5^0 + \text{⌊ . . . . . ⌋} \cdot 5^1}_{Z_3} + \text{⌊ . . . . . ⌋} \cdot 5^2}^{Z_2} + \text{⌊ . . . . . ⌋} \cdot 5^3$$

so that *natnums* $Z_k := \sum_{i \in [0\,..\,k)} t_i 5^i$ satisfy

$$f(Z_k) \equiv 0 \pmod{5^k}, \quad \text{for } k = 2, 3, 4.$$

f With $A := 23$, $B := 20$, $U := A \cdot B = 460$, let **J** be (-230 .. 230]. There is a ring-iso $F : \mathbb{Z}_A \times \mathbb{Z}_B \to \mathbb{Z}_U$ sending (α, β) to $\langle G\alpha + H\beta \rangle_U$, using magic numbers $G =$ ⌊ . . . . . ⌋ $\in$**J** and $H =$ ⌊ . . . . . ⌋ $\in$**J**. A mod-U root of poly $h(x) := 20 \cdot [x + 10]^3 + 23 \cdot [x - 2]$ is (⌊ . . . . . ⌋ , ⌊ . . . . . ⌋) $\overset{F}{\mapsto}$ ⌊ . . . . . ⌋ $\in$**J**.

g With $M := 22$ and $J := [0\,..\,M)$, use *repeated-squaring* to compute $6^{2048} \equiv_M$ ⌊ . . . . . ⌋ $\in J$. Since 2053 equals $2^{11} + 2^2 + 2^0$, the power $6^{2053} \equiv_M$ ⌊ . . . . . ⌋ $\in J$. [*Hint:* Compute with symm. residues, and use periodicity.]

OYOP: *In grammatical English* ***sentences****, write your essays on every* ***third*** *line* (usually)*, so that I can easily write between the lines. Start each essay on a* ***new*** *sheet of paper.*

C2: Prove: *If* $\mathsf{P} \equiv_4 1$ *and* P *is prime, then there exist integers* x,y *with* $x^2 + y^2 = \mathsf{P}$. [If you use melding, then give a formula for it. You may use LST without proof.]

C3: For natnums n, define

$$S_n := 13^n + 17^n + 28^n + 31^n .$$

Prove, for *odd* posints n, that S_n is composite. [*Hint:* Look at S_n mod something.]

End of Class-C

C1:	___ ___ ___	165pts
C2:	___ ___	85pts
C3:	___ ___	55pts
Total:	___ ___ ___	305pts

Folks, I have had a great time learning Number Theory with you, this Summer. Stop by in future semesters to "Talk Math".

Cheers, Prof. Erroneous Monk