Combinatorics 1 MAD4203 | **Class-C** | Prof. JLF King *Touch*: 2Apr2017

C1: Show no work. Write **DNE** in a blank <u>if</u> the described object does not exist or if the indicated operation cannot be performed.

Recall that a perm of cycle-type $\lceil 7^3, 4^2, 1^8 \rfloor$ has three 7-cycles, two 4-cycles, and eight fixed-points. Use ***CN*** for "cycle notation", and ***CCN*** for "canonical CN".

a) The number of lattice-paths from $(0,0)$ to $(31,7)$ which *never* touch the y=10 line is ________.

b) By the Binomial thm, the coeff of x^4 in $\sqrt{1+7x}$ is $\frac{n}{d}$, where integers $n \perp d$. So n= ________ and d= ________. (Write each naturally as a product of integers).

c) 2^{nd}-kind Stirling $\mathcal{S}_7(6)$= ________.

Bell number $B(4)$= ________.

d) Perm $\gamma \in \mathbb{S}_{10}$ is $(0\,1\,2\,3\,4)(5\,6\,7\,8\,9)$. The possible cycle-types of β, where $\beta^2 = \gamma$, are ________. And γ has ________ many sqroots.

e) [Orbits] Define G:[1 .. 12]↻ where $G(n)$ is the number of letters in the n^{th} Gregorian month. So $G(2) = 8$, since the 2^{nd} month is "February". The only fixed-point of G is ________. The *set* of posints k where $G^{\circ k}(12) = G^{\circ k}(7)$ is ________.

[January, February, March, April, May, June, July, August, September, October, November, December]

OYOP: *In grammatical English* ***sentences****, write your essay on every* ***third*** *line* (usually), *so that I can easily write between the lines.*

C2: i) Give a formal definition of what it means for a $\pi \in \mathbb{S}_N$ to be an ***even*** [i.e, +1] or an ***odd*** permutation [i.e, -1]. This is call the ***sign*** of π, written, $\text{Sgn}(\pi)$.

ii) Prove that every permutation in $\mathbb{S}_N$ has a well-defined sign. You will likely want to state and prove a preliminary lemma.

End of Class-C

C1:	___ ___ ___	125pts
C2:	___ ___	95pts
Total:	___ ___ ___	220pts

Please PRINT your name and ordinal. Ta:

________ Ord: ________

Honor Code: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ________