

Abstract Algebra **Class-C** Prof. JLF King  
 MAS4301 09B1 Wednesday, 20Nov2019

**C2:** Distinct elements  $\alpha, \beta \in G$  each have order 2. Prove there exists another order-2 element. [Recall “Ord( $y$ ) = 2” means  $y^2 = e$ , yet  $y \neq e$ .] [Hint: One way is to start with  $\alpha\beta\alpha$ .]

**Hi.** Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE**  $\neq \{\}$   $\neq 0 \neq$  Empty-word.

End of Class-C

**C1:** Show no work. a A *minimum* requirement for an LOR (letter-of-recommendation) from Prof. K is two courses. Circle:

Yes                      True                      Darn tootin'!

b From class, the group  $G$  of OP-isometries [Orientation-Preserving] of the cube is isomorphic to ......

Two colorings of the twelve *edges* of the cube using  $K$  colors, are *equivalent* IFF some OP-isometry carries one to the other. To compute  $\mathcal{E}(K)$ , the number of equivalence-classes, fill in this table.

What isometry $g$ ?	$\#\{\text{such } g\}$	$\#\text{Fix}(g) = K^E$ .	$E := \#\text{[Edge-orbits under } \langle g \rangle]$ .
<i>Id</i>	1	$K^{12}$	[1 <sup>12</sup> ]
FaceRot 90°			
FaceRot 180°			
VertexRot 120°			
EdgeRot 180°			

**C1:**                160pts

**C2:**                60pts

**Total:**                220pts

NAME: Ord:          

**HONOR CODE:** “I have neither requested nor received help on this exam other than from my professor.”

Signature:

And  $\mathcal{E}(K) = \left[ K^{12} + \dots \right]$ .

c A finite group  $\Gamma$  acts on a finite set  $\Omega$ . Then...  
 The number of  $\Gamma$ -orbits divides  $|\Gamma|$ : T    F  
 Cardinality of *each*  $\Gamma$ -orbit divides  $|\Gamma|$ : T    F

d In  $\mathbb{D}_8 = \langle R, F \rangle$ , the conjugacy-class of  $R^3F$  is  $\mathbb{C}(R^3F) = \{ \dots \}$  and  $\mathbb{C}(R^3) = \{ \dots \}$ .  
 The number of conjugacy-classes is ......

e Endomorphism  $f: (\mathbb{Z}_{40}, +, 0) \rightarrow (\mathbb{Z}_{40}, +, 0)$  has kernel  $\text{Ker}(f) = \{0, 8, 16, 24, 32\}$  and [this is the part I forgot to put back in]  $f(3) = 5$ . Writing  $\mathbb{Z}_{40}$  as  $[0..40)$ , then,  
 $f^{-1}(15) = \{ \dots \}$ .

OYOP: In grammatical English *sentences*, write your essay on every 2<sup>nd</sup> line (usually), so that I can easily write between the lines.