

Combinatorics
MAD4203 3214

Class-C

Prof. JLF King
Wed., 15Nov2017

Welcome. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

Use $\mathcal{S}(N, K)$ for 2Stirling #s, and use $\mathbf{c}(N, K)$ for the signless-1Stirling #s.

C1: Short answer. Show no work.

a A *minimum* requirement for an LOR (letter-of-recommendation) from Prof. K is two courses. Circle:

Yes **True** **Darn tootin'!**

b Written with \sum notation, the number of derangements of $[1..17]$ is: _____

c Let \mathbb{D}_N be the set of derangements in \mathbb{S}_N . Then $|\mathbb{D}_4| =$ _____
The set of *good* k , st. \mathbb{D}_k has both odd perms *and* even perms, is _____

d There are _____ digraphs on vertex-set $[1..N]$.
The number of *tournaments* (complete digraphs) on $[1..N]$ is _____

e For posint N , let G_N be K_N (complete graph) but with one edge removed (so G_N has N vertices). Its chromatic polynomial is $\mathcal{P}_{G_N}(x) =$ _____
[You may use rising/falling factorial in your answer, if you wish.]
In particular, $\mathcal{P}_{G_4}(x) =$ _____

C2: OYOP: In grammatical English *sentences*, write your essay on every *third* line (usually), so that I can easily write between the lines. Please number the pages "1 of 57", "2 of 57"... (or "1/57", "2/57"...). I suggest you put your name on each sheet.

i Prove that every tournament, G , on $[1..N]$ admits a **Hamiltonian** path; a path that visits each vertex exactly once (so the path will have $N-1$ edges).

ii Prove that every *DiG-connected* N -tournament, G , admits a **Hamiltonian cycle**; the cycle will have N edges.

End of Class-C

C1: 155pts

C2: 70pts

Total: 225pts

HONOR CODE: "I have neither requested nor received help on this exam other than from my professor (or his colleague)." *Name/Signature/Ord*

Ord: _____