Staple!

Ord: ___ ___

Advanced Calc
MAA4102 & MAA5104

InClass-β

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β**6:** Please carefully write up your solutions on separate sheets of paper; make sure to circle the **true/false** below. $[20 + 20 + 65 = 105$ points$]$

___ ___ A A number $\alpha \in \mathbb{R}$ is ***transcendental*** if ... [*Hint:* Do not use "algebraic" in your definition.]

___ ___ B Given a set Y, a function $\mathsf{d}: Y \times Y \to [0, \infty)$ satisfies the Triangle Inequality if... [*Hint:* Quantify all variables.]

___ ___ C Let $Y := \{0,1\}^{[1..5]}$ be the set of bit-tuples $\mathbf{b} = (b_1, b_2, b_3, b_4, b_5)$. Define $\mathsf{m}: Y \times Y \to [0, \infty)$ by: $\mathsf{m}(\mathbf{a}, \mathbf{a}) := 0$; and when $\mathbf{a} \neq \mathbf{b}$ then $\mathsf{m}(\mathbf{a}, \mathbf{b}) := j$, where j is the *smallest* (first) index at which $a_j \neq b_j$.

Circle **True** or **False**: "This function m satisfies the Triangle Inequality."

The significant part: Give a *proof* of what you claimed above.

β**7:** Please fill in the blanks. *Show no work*; there is no partial credit for this question.

i Georg Ferdinand Ludwig Philipp ________ developed the theory of (infinite) cardinals, starting his work in the 1 __ 00's.

This ________ powerset, $\mathcal{P}(\{\text{Snow White's 7 Dwarfs}\})$, has many elements.

ii Let $\mathbb{I} := \mathbb{R} \setminus \mathbb{Q}$ be the irrationals, and let $\mathbb{A}$ denote the set of algebraic numbers. Put each set, below, into its correct "cardinality pocket".
(a) $\mathbb{R} \times \mathbb{R}$. (b) $\mathbb{R}^{\mathbb{N}}$. (c) $[[\mathbb{N}^{\mathbb{N}}]^{\mathbb{N}}]^{\mathbb{N}}$. (d) $\mathbb{A}^3$. (e) $\mathbb{I}^{\mathbb{Q}}$. (f) $\mathbb{Q}^{\mathbb{I}}$. (g) $\mathbb{Q}^{\mathbb{Q}}$.

$\mathbb{N}$: ________. $\mathcal{P}(\mathbb{R})$: ________.

$\mathbb{R}$: ________. $\mathcal{P}(\mathcal{P}(\mathbb{R}))$: ________.

iii Consider $\mathbb{R}$, equipped with the usual metric.

a These *closed* sets $A_n :=$ ________, when unioned, form a set $\bigcup_{n=1}^{\infty} A_n =$ ________ which is not closed.

b Give an example of a set,

$$\{\ \_\_\_ \in \mathbb{R} \mid \_\_\_\_\_\_\_\_ \},$$

which has *exactly one* accumulation point:

iv Fix a MS (Ω, d). a For a point $p \in \Omega$, define the ball

$$\text{Ball}_{17}(p) := \{\ \_\_\_ \in \Omega \mid \_\_\_\_\_\_\_\_ \}.$$

b Suppose that $U, V_1, V_2, \ldots$ are open sets of Ω, and $E, K_1, K_2, \ldots$ are closed sets. **Circle** those of the following sets which are guaranteed to be Ω-*closed*.

$E \setminus U$. $U \setminus E$. $K_1 \setminus E$. $\bigcap_{n=1}^{\infty} K_n$.

$\Omega \setminus [\bigcup_{n=1}^{\infty} V_n]$. $E \cup K_1$. $E \cap K_1$. $[U \cap V_1]^c$.

End of InClass-β

Total: ___ 0pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

________ Ord: ___ ___