Staple!

Team B____

Sets and Logic MHF3202 17HE | **Home-B** | Prof. JLF King 16Mar2022

Due **BoC, Monday, 21Mar2022**, wATMP! Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\} \neq 0$.

Print this problem-sheet; it forms the first page of your write-up, with the blanks filled in (handwritten) and the Honor Code signed.

B1: *Show no work.*

a 15 The map $f(k, n) := 2^k \cdot [1 + 2n]$ is a bijection from $\mathbb{N}{\times}\mathbb{N} \hookrightarrow\!\!\!\!\twoheadrightarrow \mathbb{Z}_+$. And $f^{-1}(240) = ($ ______ , ______ $)$.

b 25 A $k \in [1\,..\,100]$ is ***good*** if $k \bullet 2$ or $k \bullet 3$ or $k \bullet 5$. So $^{\#}$Good = ______ . [*Hint:* Inclusion-exclusion]

c 15 15 Suppose that $\prec$ is a total-order on set $\mathcal{S}$, and $<$ is total-order on set $\boldsymbol{\Omega}$, both strict. Define binrel $\ll$ on $\mathcal{S}{\times}\boldsymbol{\Omega}$ by:

$$(b, \beta) \ll (c, \gamma)$$

IFF *Either* $b \prec c$ *or* $[b = c$ and $\beta < \gamma]$.

Then:

Relation $\ll$ *is a total-order.* T F

Suppose $\prec$ *and* $<$ *are each well-orders.*

Then $\ll$ *is a well-order.* T F

Carefully TYPE *your essays,* ***double–spaced.*** *I suggest* LaTeX*, but other systems are ok too.*

B2: For fnc $f{:}\Omega \to \Omega$, define $U_0 := \Omega$ and $U_{n+1} := f(U_n)$ and interSection

$$\mathbf{S} := \mathbf{S}_f := \bigcap_{n=0}^{\infty} U_n\,.$$

.1 Prove that $f(\mathbf{S}) \subset \mathbf{S}$.

Below, interpret $f|_{\mathbf{S}}$ as mapping $\mathbf{S} \to \mathbf{S}$ [and not as $\mathbf{S} \to \Omega$.]

.2 Prove, or give a CEX, that $f|_{\mathbf{S}}$ is *injective.*

.3 Prove, or give a CEX, that $f|_{\mathbf{S}}$ is *surjective.*

B3: [A dodecahedron is a regular polyhedron having 12 faces, 20 vertices and 30 edges; the faces are pentagons.] Two vertices of a regular dodecahedron are ***cousins*** if they are *distinct* vertices of a common face. [Each vertex has $[3 \cdot 4] - 3 = 9$ cousins.] Write $v \sim w$ to indicate that v and w are cousins. Easily, $\sim$ is symmetric, and anti-reflexive. You can check that $\sim$ is not transitive.

A ***labeling*** of a regular dodecahedron assigns, to each vertex, a *positive integer.* A labeling is ***legal*** IFF *no* pair $v \sim w$ of vertices is assigned the same label.

i Prove there is no legal labeling with vertex-sum [the sum of the 20 labels] equaling 59.

ii Let $\mathcal{S} \subset \mathbb{Z}_+$ be the *set* of vertex-sums obtainable from legal-labelings. Characterize $\mathcal{S}$ explicitly, with proof. You will likely need to construct some particular legal-labelings. [You showed, above, that $\mathcal{S} \not\ni 59$.]

B4: Produce an *interesting* Set-bijection/injection or Inclusion-Exclusion or PHP or Induction problem, then solve it (hopefully, elegantly).

B1:	___ ___	70pts
B2:	___ ___	80pts
B3:	___ ___ ___	115pts
B4:	___ ___	30pts
Total:	___ ___ ___	295pts

Honor Code: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

Ord: ______________________________

Ord: ______________________________

Ord: ______________________________