

Sets and Logic Home-B (v.g3) Prof. JLF King
 MHF3202 3E07 Wedn., 11Mar2020

Due [was: Monday, 16Mar2020, 10PM] (extended to Tuesday, 17Mar2020, by 10AM). Each team emails me (squash@ufl.edu) one PDF. The format *must* be a PDF.

Please *fill-in* every *blank* on this sheet. Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** $\neq \{\}$ $\neq 0$.

B1: *Show no work.*

a Compute the real $\alpha =$ _____ such that _____

*:
$$3^\alpha \cdot \sum_{k=0}^{4000} \binom{4000}{k} 2^k = \sum_{j=0}^{1995} \binom{1995}{j} 8^j.$$

[Hint: The Binomial Theorem]

b On $\Omega := [1..29] \times [1..29]$, define binary-relation **C** by: $(x, \alpha) \mathbf{C} (y, \beta)$ IFF $x \cdot \beta \equiv_{30} y \cdot \alpha$. Statement "*Relation C is an equivalence relation*" is:

c Circle those operators/relations which are chiral:

\neq \bullet \circ Max \div \leq $<$ \wedge

d On a K -element set, the number of reflexive symmetric binrels is _____

On a 3-set, there are _____ many equiv.relations.

For the three essay questions, carefully TYPE, double spaced, grammatical solns. I suggest L^AT_EX, but other systems are ok too.

B2: On a 7×7 chessboard, 23 rooks are placed. Prove: **Either** there exists a friendly 5-set, or a disjoint-pair of friendly 4-sets. [An n -set of rooks is **friendly** if the rooks lie on n distinct rows, and n distinct columns. Shorthand: You may use **double-clump** for "disjoint-pair of friendly 4-sets".] [Hint: PHP] **Stronger:** Prove there always exists a double-clump.

B3: Prove, for each natnum N , that

$$\sum_{k=0}^N \left[\binom{N}{k}^2 \right] = \binom{2N}{N}.$$

B4: Give a careful bijective proof of:

Thm: *Fix a natnum $N \geq 3$. Then*

*:
$$\llbracket N \downarrow 3 \rrbracket \cdot 2^{N-3} = \sum_{k=3}^N \llbracket k \downarrow 3 \rrbracket \cdot \binom{N}{k}.$$

[Can you also prove this by induction on N ?]

End of Home-B

B1: _____ 115pts

B2: _____ 125pts

B3: _____ 50pts

B4: _____ 50pts

Total: _____ 340pts

HONOR CODE: "I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)." *Name/Signature/Ord*

_____ Ord:

_____ Ord:

_____ Ord: