



Team: \_\_\_\_\_

Algebra 1  
MAS4301 3175

Home-B

Prof. JLF King  
Monday 31Mar2003

<b>B1:</b>	___ ___	40pts
<b>B2:</b>	___ ___	50pts
<b>B3:</b>	___ ___	50pts
<b>B4:</b>	___ ___	45pts
<b>B5:</b>	___ ___ ___	135pts
<b>B6:</b>	___ ___ ___	135pts
<b>Total:</b>	___ ___ ___	455pts

**Hello.** The order of your hand-in should be: PROBLEM SHEET (P-S, this side up), TYPESETTING CONVENTIONS (if any), followed by the write-up to each question. General instructions are on the CHECKLIST. *Each member* of the team must retain a *complete copy* of the team’s essays, **including diagrams**.

Use “ $f(x)$  notation” when writing fncs; in partic., for trig and log fncs. E.g, write “ $\sin(x)$ ” rather than  $\sin x$  or  $[\sin x]$ . Be sure to write unambiguously, e.g, “ $1/a + b$ ” should be bracketed either  $[1/a] + b$  or  $1/[a + b]$ .

**B1:** Please solve #22Page225. (*Jog:* The pre-image of 17.)

**B2:** Prove #24Page236. (*Jog:* The units group of a direct-sum of commutative rings.)

**B3:** Solve #49Page249. (*Jog:* Find all solns to a quadratic eqn over a finite ring.)

**B4:** Prove #5Page260. (*Jog:* Is this subring an ideal?)

**B5:** Recall, for  $N$  a posint, the dihedral group  $\mathbb{D}_N = \langle F, R \rangle$  with presentation

$$F^2 = R^N = [F \cdot R]^2 = \epsilon.$$

Draw a (large!) Cayley digraph  $\text{Cay}(\mathbb{D}_N; \{F, R\})$ .

Compute the center  $Z(\mathbb{D}_N)$ . Give a complete description –and compute the cardinality of– groups  $\text{Inn}(\mathbb{D}_N)$  and  $\text{Aut}(\mathbb{D}_N)$ . What are the orders of elements in these groups, and how many elements of each order?

**i** For what values of  $N$  can you identify (up to isomorphism) group  $\text{Inn}(\mathbb{D}_N)$ ?  $\text{Aut}(\mathbb{D}_N)$ ?

**ii** For what values of  $N$  can you identify  $\text{Out}(\mathbb{D}_N)$ , the quotient group of outer automorphisms?

**B6:** Solve the “Coconuts” problem. First solve the case where there is a coconut left-over the next morning. If the divide by 5 with one left over operation is done  $N$  times, leaving a posint amount, then what was the starting number?

**a** Find an interesting generalization of the problem, then solve it. **b** Find a version of the problem appropriate for the in-class part.

**HONOR CODE:** “I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague).” *Name/Signature/Ord*

Ord:  
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