Staple!

Team B ____

Calc 2 MAC2312 | **Home-B** | Prof. JLF King, *Touch*: 18Mar2017

Due by **3PM, Thur, 04Feb2010**. *Fill-in* every *blank* on this sheet. Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed. Do ***not*** approx.: If your result is "$\sin(\sqrt{\pi})$" then write that rather than .9797⋯. Use "$f(x)$ notation" when writing fncs; in particular, for trig and log fncs. E.g, write "$\sin(x)$" rather than the horrible $\sin x$ or $[\sin x]$.

For the essay question, TYPE ***triple–spaced***, *grammatical, solutions. Essays violate the* CHECKLIST *at Grade Peril...*

B1: In $\mathbb{R}^3$, let $\mathbf{Y}_1$ and $\mathbf{Y}_2$ be [infinite] cylinders, each of radius 1, whose axes intersect at angle $\theta \in (0, \frac{\pi}{2}]$. Use $\mathbf{Z}_1$ and $\mathbf{Z}_2$ for the corresponding *solid* cylinders. Let

$$\mathbf{B} = \mathbf{B}(\theta) := \mathbf{Z}_1 \cap \mathbf{Z}_2$$

denote "the θ-***baseball***"; their solid of intersection.

___ ___ α For several values of θ, draw LARGE carefully labeled and shaded pictures of $\mathbf{Z}_1, \mathbf{Z}_2, \mathbf{B}$. Compute the *volume* of the baseball,

$V(\theta) := \text{Vol}(\mathbf{B}(\theta)) =$ ______________.

Draw the appropriate pictures and cross-sections, carefully *explain* how you set-up the integral, then compute it. [*Hint:* You may find it useful to draw certain cross-sections of the $\mathbf{Z}_j$ before intersecting the cylinders.]

___ ___ β In the $\theta = \frac{\pi}{2}$ case, *circumscribe* a cube $\mathbf{C}$ (i.e, a regular hexahedron) about $\mathbf{B}$, and *inscribe* an octahedron $\mathbf{E}$ in $\mathbf{B}$. Compute their volumes, then show that $\text{Vol}(\mathbf{C}) > \text{Vol}(\mathbf{B}) > \text{Vol}(\mathbf{E})$.

___ ___ γ The intersection $\mathbf{Y}_1 \cap \mathbf{Y}_2$ comprises two closed curves; call the shorter curve $\mathcal{S}$ and the longer curve $\mathcal{L}$. [Well... in the $\theta = \frac{\pi}{2}$ case, the curves are congruent.] Curves $\mathcal{S}$ and $\mathcal{L}$ intersect in two points; call one of these points P.

Give a rigorous argument that $\mathcal{S}$ is a *planar* curve. [This is NOT true if the axes miss each other, or if the cylinders have different radii. So your argument will perforce use these facts in some crucial way. (Think Symmetry.)] Draw a *labeled* picture simultaneously showing $\mathbf{Y}_1, \mathbf{Y}_2, \mathcal{S}, \mathcal{L}$ and P. [If you have learned about *curvature*, what can you say about the curvature of $\mathcal{S}$ at P? And how does it change AAFOf θ?]

In an appropriate coordinate system, give an **equation** for $\mathcal{S}$. Precisely what *kind* of curve is $\mathcal{S}$, and *HDYKnow*?

___ ___ δ Compute the *surface area* of the baseball:

$\text{SA}(\mathbf{B}(\theta)) =$ ______________.

Explain the *ideas* on how you computed this; draw the appropriate pictures. [*Hint:* There is a way to do this with little computation, because a cut cylinder can be unrolled.] How do the three (compute them!) quantities $\text{SA}(\mathbf{C}), \text{SA}(\mathbf{B}), \text{SA}(\mathbf{E})$ nest? *Must* this be so? If so, why?

B2: *Henceforth, show no work. Simply fill-in each blank on the problem-sheet.*

a A delicious doughnut (DN) just fits into a box of dimensions $2W$ inches by $2W$ inches by $2H$ inches, where inequality ____ < ____ is needed for a holed-DN. Using the Thm of Pappus, how many cubic inches of delicious dough are needed to make this doughnut?

Vol= ______________ $\cdot\text{in}^3$. How many square-inches of glaze (yuck!) are needed to completely frost the doughnut? SA= ______________ $\cdot\text{in}^2$.

b $\displaystyle\int \frac{t^2}{2^t}\,dt =$ ______________. [Write ITOf $L := \log(2)$.]

c Fluid of density $D\frac{\text{lb}}{\text{ft}^3}$ fills a hemispherical tank (round side on top) of radius R ft. The work (in ft–lb) to pump it out through a hole in the top-center is

$W := \int_{\_\_\_}^{\_\_\_}$ ______________ $\cdot\, dz$.

And $W =$ ______________ $= \text{Poly}(D, R)$.

d For angles $\frac{\pi}{2} \geqslant \beta > \alpha \geqslant 0$, let $\mathbf{s}$ be the arc of the radius=1ft circle going from angle α to β. Let $f(\alpha,\beta) := \mathcal{V} + \mathcal{H}$, where $\mathcal{V}$ is the (area of the) region lying below $\mathbf{s}$ and above the x-axis, and $\mathcal{H}$ is the region horizontally between $\mathbf{s}$ and the y-axis. Integrating,

$f(\alpha,\beta) =$ ______________ $\cdot\text{ft}^2$. [*Hint:* Slice $\mathcal{V}$ vertically and $\mathcal{H}$ hor., integrating w.r.t θ. What amazing thing happens?]

End of Home-B

B1:	___ ___ ___	175pts
B2:	___ ___ ___	120pts
Un- or poorly stapled:	___	−5pts
Total:	___ ___ ___	295pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my team-mates and my professor (or his colleague)."* *Name/Signature/Ord*

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