Calc 2 MAC2312 | **Home-B** | Prof. JLF King 31Aug2015

Hello. Home-B was given out on **31 Oct., 1995**. Home-B is due, slid under my office door, by **5PM, Thursday, 2 Nov., 1995**.

B1:

a Rotating the graph of $y = \cosh(x)$, for $0 \leqslant x \leqslant 1$, about the x-axis, yield a surface of revolution of area ________.

b The graph of $y = \frac{1}{12x} + x^3$, for $x \in (1, 2)$, has length= ________.

c A delicious doughnut (**DN**) just fits into a box of dimensions 4in by 4in by 1in. Using Thm of Pappus, how many cubic inches of delicious dough are needed to make this DN? Vol= ________ in^3.

How many in^2 of glaze (yuck!) are needed to frost the doughnut? SurAr= ________ in^2.

d A certain type of bacteria increases continuously at a rate proportional to the number present. Ten hours ago there were 3,000, and now there are 12,000. In five hours there will be ________ bacteria.

e Consider the green figure (the semi-circles) of **#36(c)**[P] *533*. The *distance* from its centroid to the origin is ________.

B2: A uniform-density solid ball of radius 1 meter weighs 27 pounds. You wish to drill a hole through the center of the ball (ie., along a diameter of the ball) so that the holed-ball weighs 8 pounds. What is the **radius** of the hole you need to drill?

B3: In $\mathbb{R}^3$, let $\mathbf{Y}_1$ and $\mathbf{Y}_2$ be [infinite] cylinders, each of radius 1, whose axes intersect at angle $\theta \in (0, \frac{\pi}{2}]$. Use $\mathbf{Z}_1$ and $\mathbf{Z}_2$ for the corresponding *solid* cylinders. Let

$$\mathbf{B} = \mathbf{B}(\theta) := \mathbf{Z}_1 \cap \mathbf{Z}_2$$

denote "the θ-***baseball***"; their solid of intersection.

α For several values of θ, draw LARGE carefully labeled and shaded pictures of $\mathbf{Z}_1, \mathbf{Z}_2, \mathbf{B}$. Compute the *volume* of the baseball,

$V(\theta) := \text{Vol}(\mathbf{B}(\theta)) =$ ________.

Draw the appropriate pictures and cross-sections, carefully *explain* how you set-up the integral, then compute it. [*Hint:* You may find it useful to draw certain cross-sections of the $\mathbf{Z}_j$ <u>before</u> intersecting the cylinders.]

β In the $\theta = \frac{\pi}{2}$ case, *circumscribe* a cube **C** (i.e, a regular hexahedron) about **B**, and *inscribe* an octahedron **E** in **B**. Compute their volumes, then show that $\text{Vol}(\mathbf{C}) > \text{Vol}(\mathbf{B}) > \text{Vol}(\mathbf{E})$.

γ The intersection $\mathbf{Y}_1 \cap \mathbf{Y}_2$ comprises two closed curves; call the <u>s</u>horter curve $\mathcal{S}$ and the <u>l</u>onger curve $\mathcal{L}$. [Well... in the $\theta = \frac{\pi}{2}$ case, the curves are congruent.] Curves $\mathcal{S}$ and $\mathcal{L}$ intersect in two points; call one of these points P.

Give a rigorous argument that $\mathcal{S}$ is a *planar* curve. [This is NOT true if the axes miss each other, or if the cylinders have different radii. So your argument will perforce use these facts in some crucial way. (Think Symmetry.)] Draw a *labeled* picture simultaneously showing $\mathbf{Y}_1, \mathbf{Y}_2, \mathcal{S}, \mathcal{L}$ and P. [If you have learned about *curvature*, what can you say about the curvature of $\mathcal{S}$ at P? And how does it change AAFOf θ?]

In an appropriate coordinate system, give an <u>**equation**</u> for $\mathcal{S}$. Precisely what *kind* of curve is $\mathcal{S}$, and *HDY<u>Know</u>*?

δ Compute the *surface area* of the baseball:

$\text{SA}(\mathbf{B}(\theta)) =$ ________.

Explain the *ideas* on how you computed this; draw the appropriate pictures. [*Hint:* There is a way to do this with little computation, because a cut cylinder can be unrolled.] How do the three (compute them!) quantities $\text{SA}(\mathbf{C})$, $\text{SA}(\mathbf{B})$, $\text{SA}(\mathbf{E})$ nest? *Must* this be so? If so, why?

End of Home-B

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