Number Theory MAS4203 **Class-B** Prof. JLF King 31Aug2015

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|---|---|---|
| **B-Home:** | ___ ___ ___ | 390pts |
| **B5:** | ___ ___ | 40pts |
| **B6:** | ___ ___ | 55pts |
| **B7:** | ___ ___ | 50pts |
| **B8:** | ___ ___ | 60pts |
| **Total:** | ___ ___ ___ | 595pts |

*Please PRINT your* name and ordinal. *Ta:*

Ord: ..............................

Open brain & calculator, closed book/notes. Use $\varphi()$ for the Euler phi-fnc Essays violate the CHECKLIST at *Grade Peril*!

**B5:** Short answer: Show no work. Please write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

z) The recent deterministic polynomial-time primality test, that we will cover next semester, was done by a professor and *Circle*:
**Two gorillas Two undergrads Twin primes Simon&Garfunkel**

b) Compute *Magic integers* $C_1$= ______, $C_2$= ______, $C_3$= ______, each in $[-165\,..\,165]$, so that mapping $g\colon \mathbb{Z}_6 \times \mathbb{Z}_5 \times \mathbb{Z}_{11} \to \mathbb{Z}_{330}$ is a ring-isomorphism, where

$$g\big((z_1, z_2, z_3)\big) := \Big\langle z_1 C_1 + z_2 C_2 + z_3 C_3 \Big\rangle_{330}.$$

Verify that your map satsifies: $g\big((1,1,1)\big) = 1$ and $g\big((z+6,0,0)\big) = g\big((z,0,0)\big)$ and etc..

*Essay questions: Write in complete sentences and also fill-in the blanks:*

**B6:** Let $f(x) := x^2 - 4x - 2$ and $z_0 := c_0 := 1$. Note $f(z_0) \equiv_5 0$. Note $f'(z_0)$= ______ $\not\equiv_5 0$.

Use Hensel's lemma repeatedly to compute coefficients $c_k \in [-2\,..\,2]$ (these are the blanks, below)

$$z_3 = \underbrace{\overbrace{c_0 \cdot 5^0 + \underline{\quad} \cdot 5^1}^{z_1} + \underline{\quad} \cdot 5^2}_{z_2} + \underline{\quad} \cdot 5^3$$

so that integers $z_k := \sum_{i=0}^{k} c_k 5^i$ satisfy

$$f(z_k) \equiv_{5^{k+1}} 0,$$

for $k = 1, 2, 3$.

**B7:** For a set $M$, a mapping $\mathsf{d}\colon M \times M \to [0, \infty)$ is a ***metric*** if.... It is an ***ultrametric*** if the Triangle Inequality can be strengthened to...

A sequence $\mathbf{s} := (m_j)_1^\infty$ in $M$ is $\mathsf{d}$-***Cauchy*** if...

**B8:** Consider posints $T,G,N$ with $T \perp G$. Use CRT (and not Dirichlet's thm) to prove that there exists $z \in T + G\mathbb{Z}$ with $z \perp N$. If desired, you may assume that the prime factorization of $N$ is $N = \mathsf{p}^b \cdot \mathsf{q}^c$, where $\mathsf{p} \bullet\!| \, G$ and $\mathsf{q} \nmid G$.

No RONOs were harmed in the making of this exam.

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ..............................