

Sets and Logic
MHF3202 17HE

Class-B

Prof. JLF King
Wednesday, 23Mar2022

B5: Short answer. Show no work.

Write **DNE** if the object does not exist or the operation cannot be performed. NB: **DNE** \neq $\{\}$ \neq 0.

a Fall 2022, Prof. K teaches ABSTRACT ALGEBRA, Lit235, 7th (13:55-14:45). **True** **Yes** Who's Al Gebra?

b A bijection $f: [5, 6] \leftrightarrow (0, 1)$ is: $f(5) :=$ _____ ;
 $f($ _____ $) :=$ _____ , for each $k \in$ _____ ;
 and $f(x) :=$ _____ , for each $x \in (5, 6] \setminus C$,
 where $C :=$ _____ .

c For a finite list \mathcal{S} of posints, define

$$\mu_{\mathcal{S}}(N) := \{k \in [1..N] \mid \exists d \in \mathcal{S} \text{ with } d \mid k\}.$$

Use Inclusion-Exclusion to compute

$$|\mu_{\{6,7,9\}}(67)| = \text{_____}$$

d Written with \sum notation, the number of derangements of $[1..17]$ is: _____

OYOP: In grammatical English *sentences*, write your essays on every 2nd line (usually), so I can easily write between the lines.

B6: Interval-of-integers $\mathbf{J} := [201..300]$ has 99 elements. A subset $S \subset \mathbf{J}$ is **Big** if $|S| = 51$. Subset $S \subset \mathbf{J}$ is **Perfect** if there exist *distinct* members $x, y \in S$ st. $x + y = 500$.

Prove that **Big** \Rightarrow **Perfect**. [Hint: PHP. Carefully specify what your pigeon-holes are.]

B7: For $K = 0, 1, 2, \dots$, define sum

$$S_K := \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{K \cdot [K+1]}$$

$$\stackrel{\text{note}}{=} \sum_{n=1}^K \frac{1}{n \cdot [n+1]}.$$

Find a closed-form [no summation sign, nor dot-dot-dot] for S_K . Prove your formula correct by induction on K .

B5: _____ 80pts

B6: _____ 55pts

B7: _____ 50pts

Total: _____ 185pts

NAME: _____

HONOR CODE: "I have neither requested nor received help on this exam other than from my professor."

Signature: _____