Start: ________ **Stop:** ________ **Name:** ____________________

Sets and Logic MHF3202 | **Online-B** | Prof. JLF King Fri, 06Nov2020

B1: Short answer. Show no work. 65 points, total.

20 **a** Compute the real $\alpha=$ ________ such that

$$*: \qquad 3^{\alpha}\cdot\sum_{k=0}^{4000}\binom{4000}{k}2^{k} \quad = \quad \sum_{j=0}^{1995}\binom{1995}{j}8^{j}\,.$$

[*Hint:* The Binomial Theorem]

25 20 **b** For a finite list $\mathcal{S}$ of posints, define

$$\boldsymbol{\mu}_{\mathcal{S}}(N) \;:=\; \Big\{k\in[1\,..\,N] \;\Big|\; \exists\, d\in\mathcal{S} \;\; with \;\; d \bullet\!| \; k\Big\}.$$

For $\mathcal{S}:=\{6,7,10\}$, the Inclusion-Exclusion formula (using the floor fnc) for the number of elements, $|\boldsymbol{\mu}_{\mathcal{S}}(N)|$, is

____

(Write your answer, using the floor function as appropriate, in form [term + term + term] − [term + term + term] + term. The terms are computed from the $\{6,7,10\}$ numbers.)

When $N:=67$, then, $|\mu_{\{6,7,10\}}(67)| =$ ________.

B2: Short answer. Show no work. 95 points, total.

15 15 **c** Number $\left[\mathbf{i}+\sqrt{3}\right]^{70} = x+\mathbf{i}y$, for real numbers $x=$ ________ and $y=$ ________.

[*Hint: Multiplying complexes multiplies their moduli, and adds their angles.*]

10 10 10 5 **d** Blanks∈ℝ. So $\frac{1}{2+3\mathbf{i}}=$ ________ $+\mathbf{i}\cdot$[________].

Thus $\mathrm{Im}\left(\dfrac{5-\mathbf{i}}{2+3\mathbf{i}}\right) =$ ________.

By the way, $|5-3\mathbf{i}| =$ ________.

15 15 **e** The ***Threeish-numbers*** comprise $\boldsymbol{\mathcal{T}} := 1+3\mathbb{N}$.

$\boldsymbol{\mathcal{T}}$-number $385 \stackrel{\text{note}}{=\!=} 35\cdot 11$ is $\boldsymbol{\mathcal{T}}$-*irreducible*: T F

Threeish $N:=85$ is **not** $\boldsymbol{\mathcal{T}}$-*prime* because $\boldsymbol{\mathcal{T}}$-numbers $J:=$ ________ and $K:=$ ________ satisfy that $N \bullet\!| \;[J\cdot K]$, **yet** $N \not\bullet\!| \; J$ and $N \not\bullet\!| \; K$.

B3: Short answer. Show no work. 70 points, total.

5 5 5 5 **f** Consider binrels on $\Omega :=$ Stooges $:= \{M, L, C\}$. There are ________ **Anti-reflexive** binrels, and ________ **Reflexive** binrels, and ________ **Symmetric** binrels. The number of **strict total-orders** is ________.

20 **g** On $\Omega := [1\,..\,29]\times[1\,..\,29]$, define binary-relation $\mathbf{C}$ by: $(x,\alpha)\,\mathbf{C}\,(y,\beta)$ IFF $x\cdot\beta \equiv_{30} y\cdot\alpha$. Statement "*Relation* $\mathbf{C}$ *is an* ***equivalence relation***" is: T F

10 **h** Let $\mathcal{P}_{\infty}$ denote the family of all *infinite* subsets of $\mathbb{N}$. Define relation $\approx$ on $\mathcal{P}_{\infty}$ by: $A\approx B$ IFF $A\cap B$ is infinite. Stmt "*This* $\approx$ *is an equivalence-relation*" is: T F

10 10 **i** Both $\sim$ and $\bowtie$ are equiv-relations on a set Ω. Define binrels $\mathbf{I}$ and $\mathbf{U}$ on Ω as follows.

Define $\omega\,\mathbf{U}\,\lambda$ IFF <u>*Either*</u> $\omega\sim\lambda$ <u>*or*</u> $\omega\bowtie\lambda$ [or both].

Define $\omega\,\mathbf{I}\,\lambda$ IFF <u>*Both*</u> $\omega\sim\lambda$ <u>*and*</u> $\omega\bowtie\lambda$.

So "$\mathbf{U}$ *is an equiv-relation*" is: T F

So "$\mathbf{I}$ *is an equiv-relation*" is: T F

B1:	___ ___	65pts
B2:	___ ___	95pts
B3:	___ ___	70pts
Total:	___ ___ ___	230pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ________________________________