

Sets and Logic
MHF3202 2787

Class-B

Prof. JLF King
Wednesday, 24Oct2018

B4: Short answer. Show no work.

a Prof. King's Number Theory and Mathematical Cryptography course will be offered 7th period [1:55 PM], next semester, Spring 2019. Circle:

Yes

True

Mais oui!

b From the 195×160 game-board, cut-out (remove) the (99, 27)-cell and one other cell at $P = (x, y)$. Circle those choices for P ,

(160, 150), (124, 5), (76, 67), (194, 159), (51, 7)

which, if removed, would leave a board that *definitely cannot* be domino-tiled. Let \equiv mean \equiv_2 . Color the (x, y) cell Blue, if $x \equiv y$; color it Red if $x \not\equiv y$. The Removed cell is Blue. Removing another Blue cell guarantees that the board is untileable, since each domino covers one Blue and one Red cell. **Whoa! Why?** Well, the board started with an *equal number* of each color [HDYKnow?], since 160 is even.

c On \mathbb{R}_+ , define several relations: Say that xRy IFF $y - x < 17$. Define \mathcal{P} by: xPy IFF $x^{\log(y)} = 5$.

Say that xIy IFF $x + y$ is irrational.

Use \blacklozenge for the "divides" relation on the positive integers: $k \blacklozenge n$ iff there exists a posint r with $rk = n$.

c1 Please circle those of the following relations which are *transitive* (on their domain of defn).

\neq \leq \mathcal{R} \mathcal{P} \mathcal{I}

c2 Circle the *symmetric* relations:

\neq \blacklozenge \leq \mathcal{R} \mathcal{P} \mathcal{I}

c3 Circle the *reflexive* relations:

\neq \leq \mathcal{R} \mathcal{P} \mathcal{I}


d A $k \in [1..100]$ is *good* if $k \blacklozenge 2$ or $k \blacklozenge 3$ or $k \blacklozenge 5$. So #Good = 74. [Hint: Inclusion-exclusion]

Soln: On interval $[1..T]$, the number of multiples of posint N is $\lfloor \frac{T}{N} \rfloor$. The number of tokens which are simultaneously multiples of J and K is $\lfloor \frac{T}{\text{LCM}(J,K)} \rfloor$.

e A *region* is a connected finite union of unit-squares in the plane. Regions B and C are disjoint; let $U := B \sqcup C$. Let **Til**="Lmino tilable" and **Not**="Not Lmino tilable".

$[B \text{ Til and } C \text{ Not}] \implies [U \text{ Not}]$ $\begin{matrix} T & \textcircled{F} \\ T & \textcircled{F} \\ T & \textcircled{F} \end{matrix}$
 $[B \text{ Not and } C \text{ Not}] \implies [U \text{ Not}]$
 $[B \text{ Til and } C \text{ Til}] \implies [U \text{ Til}]$ $\begin{matrix} T & \textcircled{F} \\ T & \textcircled{F} \\ T & \textcircled{F} \end{matrix}$

OYOP: In *grammatical English sentences*, write your essay on every 2nd or 3rd line (usually), so that I can easily write between the lines. Please number the pages "1 of 3", "2 of 3"....

B5: An *Lmino* (pron. "ell-mino") comprises three \blacksquare squares in an "L" shape (all four orientations are allowed). For natnum N , let \mathbf{R}_N denote the $3 \times N$ board: I.e.,  is the \mathbf{R}_5 board. Prove:

Theorem: When N is odd, then board \mathbf{R}_N is not Lmino-tilable.

You will likely want to first *state* and *prove* a Lemma. Now use appropriate induction on N to prove the thm. Also: *Illustrate your proof* with (probably several) large, *labeled* pictures.

Also, for $N=2H$ *even*, \mathbf{R}_N has 2^H many Lmino-tilings (with proof).

B4: 121pts

B5: 50pts

Total: 171pts

NAME: Ord:

HONOR CODE: "I have neither requested nor received help on this exam other than from my professor."

Signature: