Staple!

Ord: ___ ___

Sets and Logic
MHF3202 2787

Class-B

Prof. JLF King
Wednesday, 24Oct2018

B4: Short answer. Show no work.

__ __ **a** Prof. King's Number Theory and Mathematical Cryptography course will be offered 7^{th} period [1:55 PM], next semester, Spring 2019. Circle:

Yes **True** ***Mais oui!***

__ __ **b** From the 195×160 game-board, cut-out (remove) the **(**99, 27**)**-cell and one other cell at $P = (x, y)$. Circle those choices for P,

$(160, 150)$, $(124, 5)$, $(76, 67)$, $(194, 159)$, $(51, 7)$

which, if removed, would leave a board that *definitely* can**not** be domino-tiled.

__ __ **c** *On* $\mathbb{R}_+$, *define several relations: Say that* $x\mathcal{R}y$ *IFF* $y - x < 17$. *Define* $\mathcal{P}$ *by:* $x\mathcal{P}y$ *IFF* $x^{\log(y)} = 5$.
Say that $x\mathcal{I}y$ *IFF* $x + y$ *is irrational.*
Use $\bullet\!|$ *for the "divides" relation on the positive integers:* $k \bullet\!| n$ *iff there exists a posint* r *with* $rk = n$.

c1 Please circle those of the following relations which are *transitive* (on their domain of defn).

$\neq$ $\quad \bullet\!|$ $\quad \leqslant$ $\quad \mathcal{R}$ $\quad \mathcal{P}$ $\quad \mathcal{I}$

c2 Circle the *symmetric* relations:

$\neq$ $\quad \bullet\!|$ $\quad \leqslant$ $\quad \mathcal{R}$ $\quad \mathcal{P}$ $\quad \mathcal{I}$

c3 Circle the *reflexive* relations:

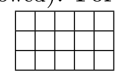
$\neq$ $\quad \bullet\!|$ $\quad \leqslant$ $\quad \mathcal{R}$ $\quad \mathcal{P}$ $\quad \mathcal{I}$

__ __ **d** A $k \in [1..100]$ is ***good*** if $k \not\bullet\!| 2$ or $k \not\bullet\!| 3$ or $k \not\bullet\!| 5$. So #Good = __________. [*Hint:* Inclusion-exclusion]

__ __ **e** A ***region*** is a connected finite union of unit-squares in the plane. Regions B and C are disjoint; let $U := B \sqcup C$.
Let **Til**="Lmino tilable" and **Not**="Not Lmino tilable".

[B **Til** and C **Not**] $\implies$ [U **Not**]	*T*	*F*
[B **Not** and C **Not**] $\implies$ [U **Not**]	*T*	*F*
[B **Til** and C **Til**] $\implies$ [U **Til**]	*T*	*F*

OYOP: *In grammatical English sentences, write your essay on every* 2^{nd} *or* 3^{rd} *line* (usually), *so that I can easily write between the lines.* Please number the pages "1 of 3", "2 of 3"....

B5: An ***Lmino*** (pron. "ell-mino") comprises three  squares in an "L" shape (all four orientations are allowed). For natnum N, let $\mathbf{R}_N$ denote the $3 \times N$ board: I.e,  is the $\mathbf{R}_5$ board. Prove:

Theorem: When N *is odd, then board* $\mathbf{R}_N$ *is* *<u>not</u> Lmino-tilable.*

You will likely want to first *state* and *prove* a Lemma. Now use appropriate induction on N to prove the thm. Also: *Illustrate your proof* with (probably several) large, *<u>labeled</u>* pictures.
Also, for $N = 2H$ *<u>even</u>*, $\mathbf{R}_N$ has __________ many Lmino-tilings (with proof).

B4:	__ __ __	121pts
B5:	__ __	50pts
Total:	__ __ __	171pts

NAME: ________________________ Ord: ______

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor."*

Signature: ________________________________