Plex
MAA4402 2838

Class-B

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Abbrevs. Let ***SCC*** mean "positively oriented simple-closed-contour". For a SCC C, have $\mathring{C}$ be the (open) region C encloses, and let $\widehat{C}$ mean C together with $\mathring{C}$. So $\widehat{C}$ is $C \cup \mathring{C}$; it is automatically simply-connected and is a closed bounded set.

Use P.V. for "principal value", and Log() for P.V of logarithm. Use ln() for natural logarithm.

Let U be SCC $\mathrm{Sph}_3(\boldsymbol{i})$, a circle of radius 3.

B1: Short answer. Show no work.

Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

z) The visual representation of $\mathbb{C}$ is sometimes called "the ? plane", where ? is Circle: **Air Sea De Higher Unreal Snakes-on-a Argand Krypton Radon Xenon Euler Please-x y-com Rain-in-Spain-stays-mainly-on-the** .

a) Define $f(x+\boldsymbol{i}y) := xy + \boldsymbol{i}x$. Let L be the line-segment from the origin to $2+\boldsymbol{i}$. Then $\int_L f(z)\,\mathrm{d}z =$ ________.

b) Value $\oint_U \frac{\mathrm{e}^{3z}}{[z-2]^5}\,\mathrm{d}z =$ ________

[Answer may be written as a product, using powers and factorials.]

c) Integral $\oint_U \frac{\cos(z^2)}{3-z}\,\mathrm{d}z =$ ________,

and $\oint_U \frac{\cos(z^2)}{1-3z}\,\mathrm{d}z =$ ________.

[*Hint:* Does Cauchy Integral Formula apply? Cauchy-Goursat?]

d) Value $\mathrm{Log}([\boldsymbol{i}\mathrm{e}]^3) =$ ________.

[P.V of $[1+\boldsymbol{i}]^{\boldsymbol{i}}$] $= r\cdot\exp(\boldsymbol{i}\theta)$, where $r =$ ________ and $\theta =$ ________, with $r>0$ and θ real.

e) Coeff of x^3y^6 in $[x+5y]^9$ is ________.

[Write your answer as a product of powers and factorials.]

OYOP: *In grammatical English* ***sentences****, write your essay on every* ***third*** *line* (usually)*, so that I can easily write between the lines.* Start each essay on a new sheet-of-paper. Please number the pages "1 of 57", "2 of 57"... (or "1/57", "2/57"...) I suggest you put your name on each sheet.

B2: Consider a domain $D \subset \mathbb{C}$ and a continuous fnc $h{:}\, D \to \mathbb{C}$ with the ***Path Independence Property*** [***PIP***]:

> *Each two contours $C_1, C_2 \subset D$ that start at the same point, and end at the same point, satisfy that* $\int_{C_1} h(z)\,\mathrm{d}z = \int_{C_2} h(z)\,\mathrm{d}z$.

Prove there exists a differentiable function $g{:}\, D \to \mathbb{C}$, with $g' = h$.

B1:	___ ___ ___	175pts
B2:	___ ___	60pts
Total:	___ ___ ___	235pts

HONOR CODE: *"I have neither requested nor received help on this exam other than from my professor (or his colleague)."*
Name/Signature/Ord

Ord: ________