

Abbrevs. Let **SCC** mean “positively oriented simple-closed-contour”. For a **SCC** C , have $\overset{\circ}{C}$ be the (open) region C encloses, and let \widehat{C} mean C together with $\overset{\circ}{C}$. So \widehat{C} is $C \cup \overset{\circ}{C}$; it is automatically simply-connected and is a closed bounded set.

Use P.V. for “principal value”, and $\text{Log}()$ for P.V. of logarithm. Use $\ln()$ for natural logarithm.

Let U be **SCC** $\text{Sph}_3(i)$, a circle of radius 3.

B1: Short answer. Show no work.

Write **DNE** in a blank if the described object does not exist or if the indicated operation cannot be performed.

z The visual representation of \mathbb{C} is sometimes called “the ? plane”, where ? is Circle: **Air Sea De Higher Unreal Snakes-on-a Argand Krypton Radon Xenon Euler Please-x y-com Rain-in-Spain-stays-mainly-on-the .**

a Define $f(x + iy) := xy + ix$. Let L be the line-segment from the origin to $2+i$. Then $\int_L f(z) dz =$ _____.

b Value $\oint_U \frac{e^{3z}}{[z-2]^5} dz =$ _____
[Answer may be written as a product, using powers and factorials.]

c Integral $\oint_U \frac{\cos(z^2)}{3-z} dz =$ _____,

and $\oint_U \frac{\cos(z^2)}{1-3z} dz =$ _____.

[Hint: Does Cauchy Integral Formula apply? Cauchy-Goursat?]

d Value $\text{Log}([ie]^3) =$ _____.

[P.V. of $[1 + i]^i$] = $r \cdot \exp(i\theta)$, where $r =$ _____ and $\theta =$ _____, with $r > 0$ and θ real.

e Coeff of x^3y^6 in $[x + 5y]^9$ is _____.

[Write your answer as a product of powers and factorials.]

OYOP: In grammatical English sentences, write your essay on every *third* line (usually), so that I can easily write between the lines. Start each essay on a new sheet-of-paper. Please number the pages “1 of 57”, “2 of 57”... (or “1/57”, “2/57”...) I suggest you put your name on each sheet.

B2: Consider a domain $D \subset \mathbb{C}$ and a continuous fnc $h: D \rightarrow \mathbb{C}$ with the **Path Independence Property [PIP]**:

Each two contours $C_1, C_2 \subset D$ that start at the same point, and end at the same point, satisfy that $\int_{C_1} h(z) dz = \int_{C_2} h(z) dz$.

Prove there exists a differentiable function $g: D \rightarrow \mathbb{C}$, with $g' = h$.

B1: _____ 175pts

B2: _____ 60pts

Total: _____ 235pts

HONOR CODE: “I have neither requested nor received help on this exam other than from my professor (or his colleague).”
Name/Signature/Ord

Ord: _____